A Comparison of Various Algorithms for the Spectral Analysis of Unevenly Spaced Data Series

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The Problem

Interpolation Approach

Least Squares Approach

Hybrid Approach

Comparisons

Conclusions

The Problem - Spectral Analysis

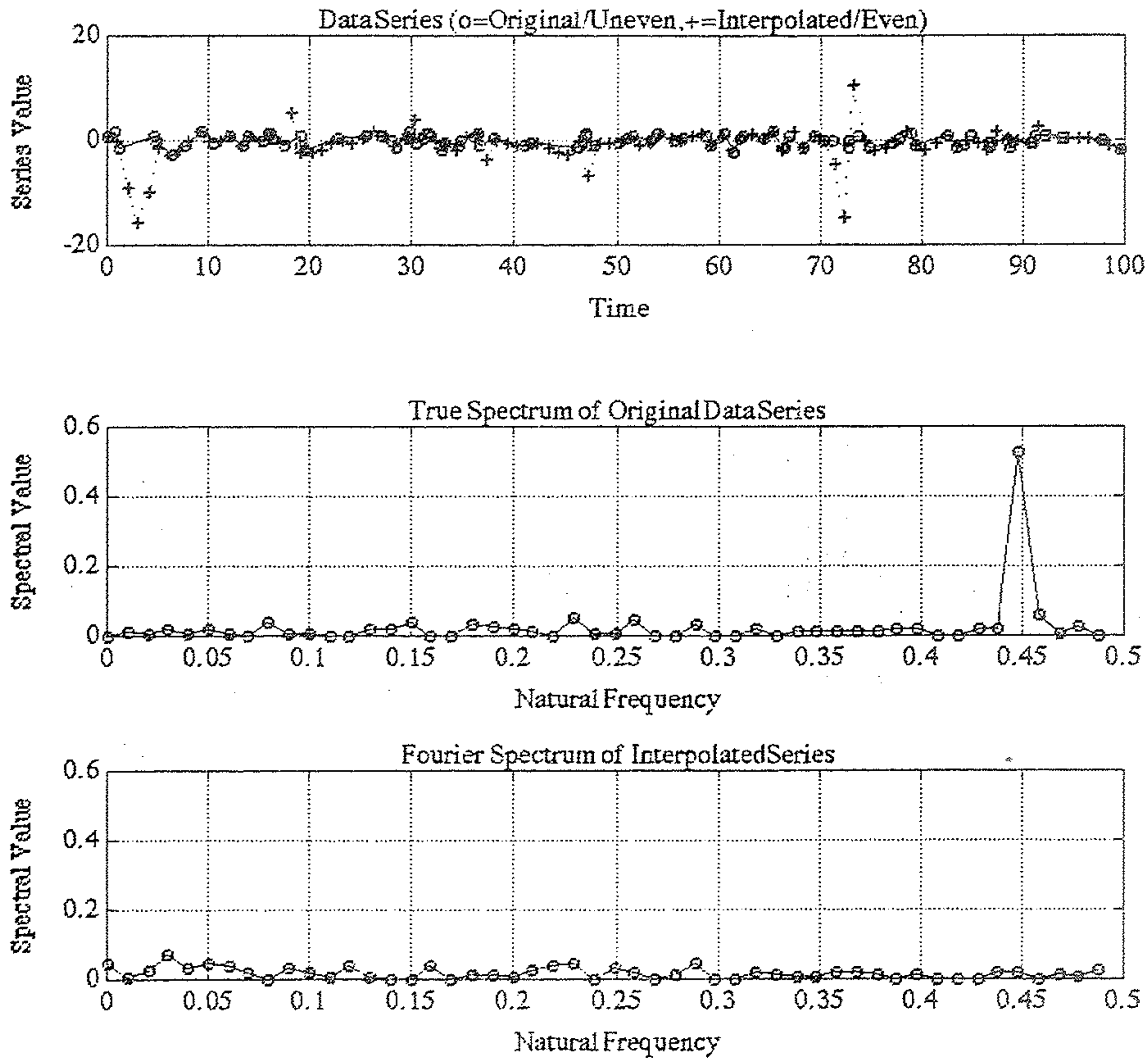
- Uneven data spacing
  - » Missing data points (gaps)
  - » Randomly spaced series
- Presence of signals
  - » Datum biases, linear trends, known periodic trends (tides)
  - » Should be estimated along with spectral analysis
- Fourier spectral analysis algorithms require
  - » Evenly spaced data series
  - » Integer multiples of the fundamental frequency (discrete)
  - » 2^n data points (FFT algorithm only)
  - » No interaction (correlation) between estimated signals and spectral values
  - » Doesn't account for finiteness of data (assumes infinite length series)

Interpolation Approach

- Interpolation to evenly spaced series (creates new data series)
- FFT of interpolated (evenly spaced) data series
- Accuracy depends on
  - » Form of interpolation function
  - » Smoothness of original series
  - » Presence of large data gaps

Problems

- Spectrum for interpolated data series, <u>not original series</u>
- Interpolation
  - » Smooths high frequency part of data series
  - » No way of assessing accuracy
  - » Very poor in presence of data gaps
- FFT
  - » Most algorithms limited to 2^n data points
  - » Only "integer frequencies" (discrete spec.)
  - » Can't zoom in to precisely locate new peaks


DataSeries (o=Original/Uneven,+=Interpolated/Even)
Series Value
20
0
-20
0
10
20
30
40
50
60
70
80
90
100
Time
True Spectrum of Original DataSeries
Spectral Value
0.6
0.4
0.2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Natural Frequency
Fourier Spectrum of InterpolatedSeries
Spectral Value
0.6
0.4
0.2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Natural Frequency

Least Squares Spectral Analysis (Vaníček)

- Direct least squares fitting of Fourier series terms to data
- Spectral values = relative variance explained by fitting each frequency (percent of total variance explained)
- Provides spectral values at any frequency - *continuous*
- Correlation among observations - include a weight matrix
- Presence of known signals - include additional functions when fitting each frequency
- Fourier spectrum just a special case
    - » Evenly spaced data
    - » "Integer" frequencies (discrete spectrum)

- Simultaneous evaluation of all Fourier terms (all at once)

  » Can result in singularities for some frequencies
  » Requires A LOT OF RAM
  » Very slow

- Separate evaluation of Fourier terms (one at a time) - LSSA

  » Avoids singularities
  » Little memory required
  » Faster
  » Significant spectral frequencies estimated simultaneously as signals to be removed

Advantages

- Correct spectrum - not affected by finiteness of data and uneven spacing
- Spectral values for any frequencies (continuous spectrum) - => Can zoom in on peaks
- Accounts for interaction (correlation) of known signals with spectral frequencies

LSSA Algorithms

- Based on different numerical procedures

- Direct inversion of normal equations (Vaníček, 1969) - LSSA
  - » Most general
  - » Accounts for presence of estimated signals

- Orthogonalization via time shifting (Lomb, 1975)
  - » Shift time so that normal eqns are diagonal
  - » Different time shift for each frequency [redacted].
  - » Doesn't account for estimated signals
  - » Slower than direct inversion

- Gram-Schmidt orthogonalization (Ferraz-Mello, 1981)
  - » Doesn't account for estimated signals

- All are mathematically equivalent under same assumptions (no estimated signals)

Hybrid FFT-LSSA Approach (Press & Rybicki)

- Based on "extirpolation" & FFT - FASPER (very fast)
- Extirpolation
  - » Find an equally spaced series that gives the original unequally spaced one after interpolation (reverse interpolation)
- FFT
  - » Used to evaluate evenly spaced (extirpolated) sine and cosine summations
  - » Applied to time-shifting algorithm
  - » Original algorithm requires two complex FFTs
  - » Can be reduced to only one FFT using a trigonometric identity

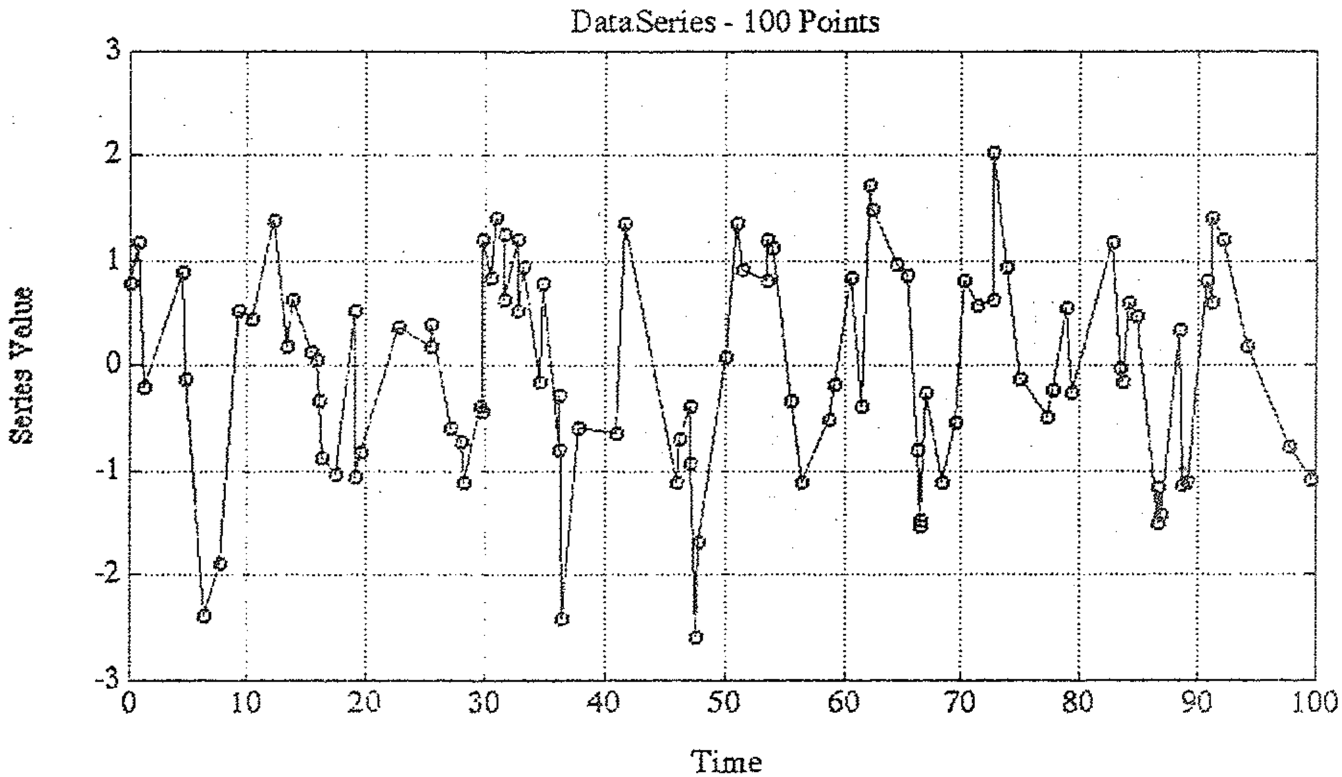
Disadvantages

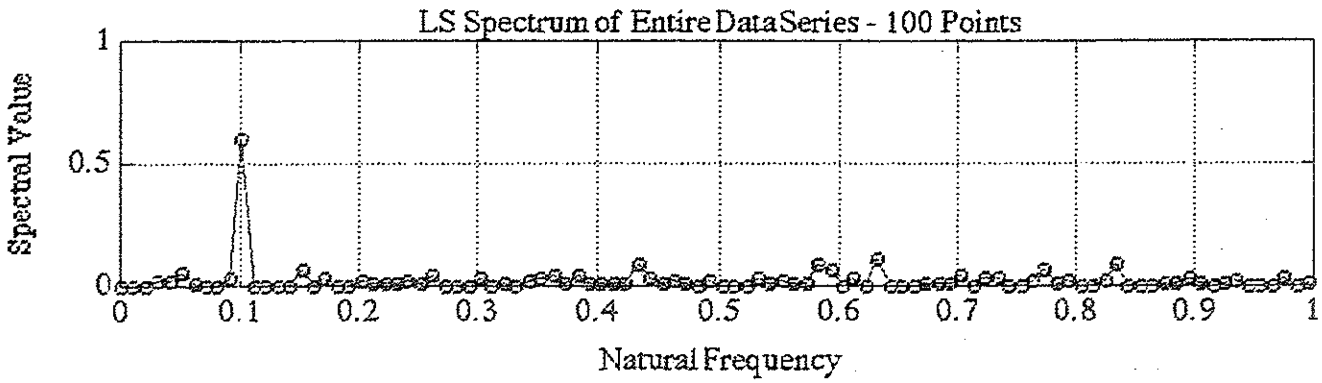
- » Extirpolation accuracy unknown
- » Same as FFT (2^n data points, "integer" frequencies)
- » Doesn't account for estimated signals

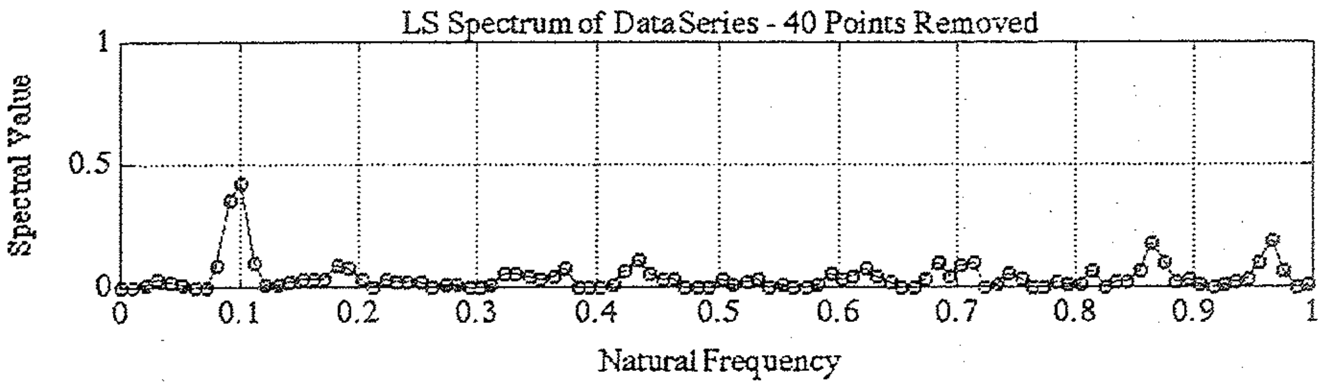
Example

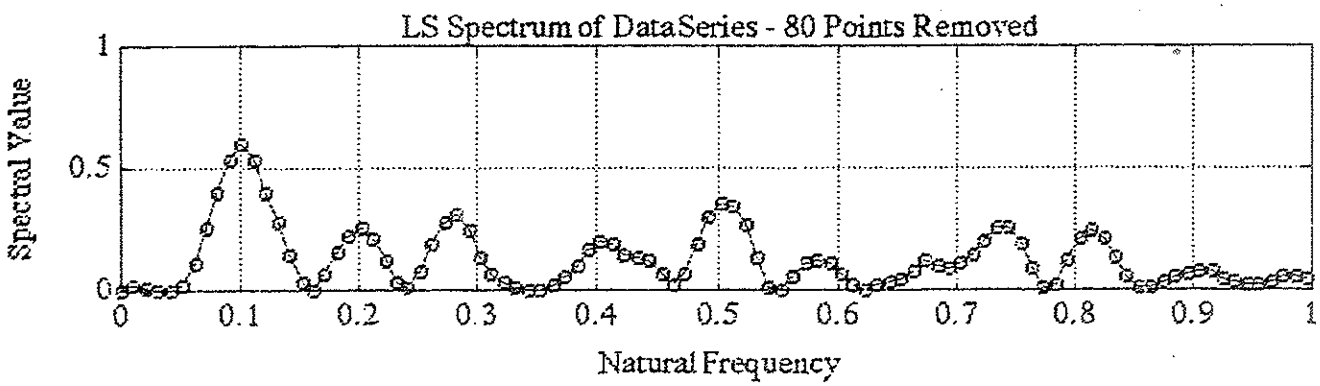
Periodic signal
- » Period = 10
- » Magnitude = ±1

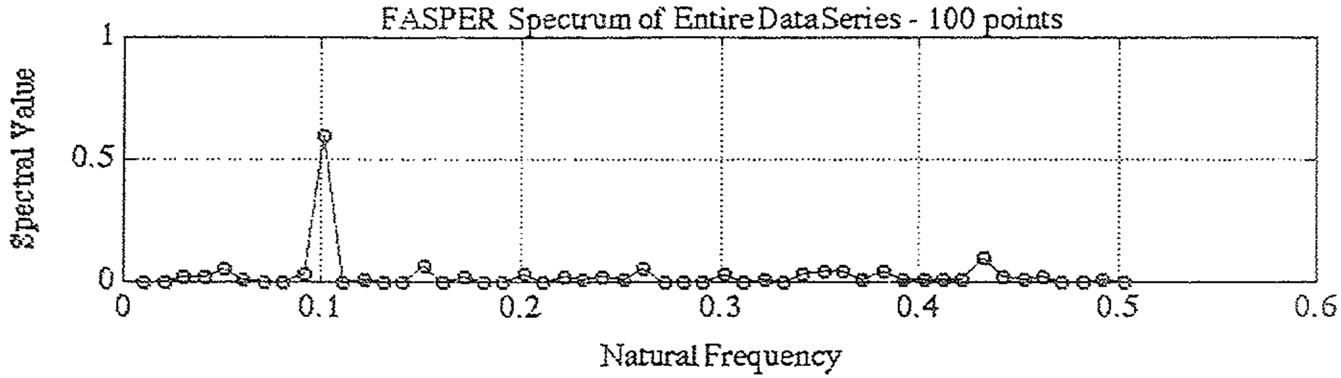
Pseudo-random noise
- » Mean = 0
- » Standard deviation = 2/3

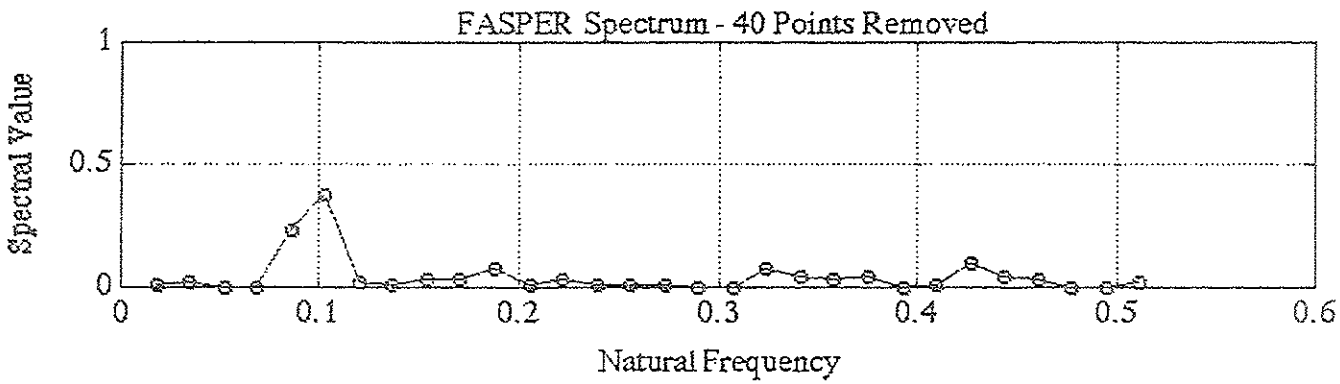


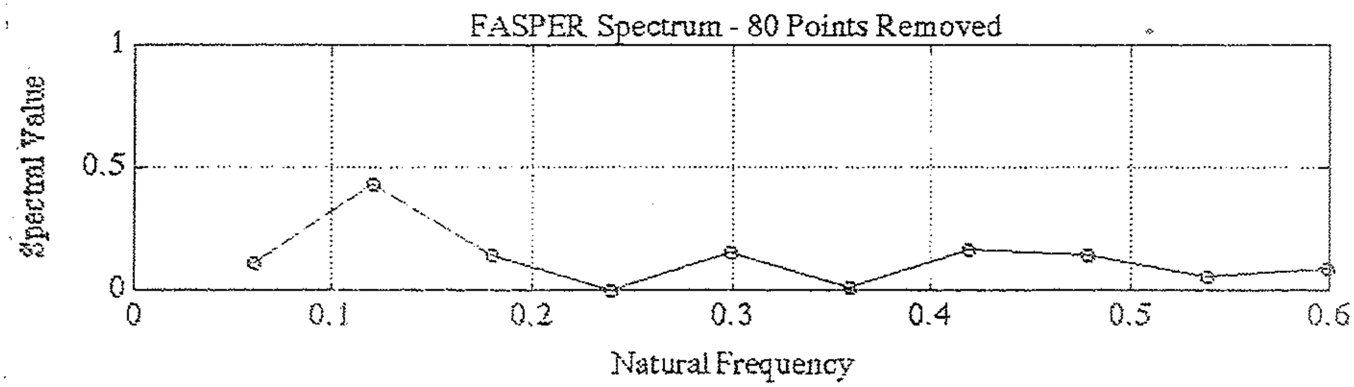

LS Spectrum of Entire DataSeries - 100 Points
Spectral Value
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Natural Frequency


LS Spectrum of DataSeries - 40 Points Removed
Spectral Value
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Natural Frequency


LS Spectrum of DataSeries - 80 Points Removed
Spectral Value
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Natural Frequency


FASPER Spectrum of Entire Data Series - 100 points
Spectral Value
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
Natural Frequency


FASPER Spectrum - 40 Points Removed
Spectral Value
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
Natural Frequency


FASPER Spectrum - 80 Points Removed
Spectral Value
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
Natural Frequency

Conclusions

- LSSA
  - » Best results - most reliable
  - » Provides a *continuous spectrum*
  - » Accounts for presence of estimated signals
  - » Relative slow
- FASPER
  - » Very fast (especially with extra trig. identity)
  - » Must be very careful - accuracy of extirpolation unknown
  - » Provides only a *discrete spectrum*
  - » More research needed to ascertain limitations
- Interpolation - Very poor => Forget it!

Recommend

- » Use LSSA whenever possible
- » Exercise great caution when using FASPER
- » At least use LSSA as a check on FASPER

Software

- LSSA

  » Wells, Vaníček and Pagiatakas (1985). Least Squares Spectral Analysis Revisited Surveying Engineering Technical Report No. 84.

  » Available for Macintosh, IBM PC and mainframe

- FASPER

  » Press and Rybicki (1989). Fast algorithm for spectral analysis of unevenly sampled data. The Astronomical Journal, Vol. 338, pp. 277-280.

  » Paper contains FASPER routine

  » Uses some Numerical Recipes routines