ROBUSTNESS ANALYSIS

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September 1991

TECHNICAL REPORT
NO. 156

PREFACE

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ROBUSTNESS ANALYSIS

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November 1990
Corrected March 1991
Latest Reprinting April 1996

PREFACE

This technical report is a reproduction of a final contract report submitted to Geodetic Survey Division, Canada Centre for Surveying, Energy, Mines and Resources Canada, November 1990.

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Vaníček, P., E.J. Krakiwsky, M.R. Craymer, Y. Gao, and P.S. Ong (1990). "Robustness analysis." Final contract report, Department of Surveying Engineering Technical Report No. 156, University of New Brunswick, Fredericton, New Brunswick, Canada, 116 pp.

EXECUTIVE SUMMARY

Described in this report are the results of the investigations undertaken by two collaborating research groups at the University of New Brunswick and The University of Calgary under a DSS research contract #23244-9-4198/01-SS for the Geodetic Survey Division of the Canada Centre for Surveying. The investigations addressed the problem of geodetic network analysis techniques, and proposed alternatives to the standard statistical analysis techniques designed to analyse network sensitivity to gross errors and blunders.

The original aim of the investigations was to study the differences between and merits of two such alternative techniques: the reliability technique, introduced by Baarda and implemented by The University of Calgary group, and the geometrical strength analysis formulated by the University of New Brunswick group. It was discovered at the beginning of the investigation that these two techniques are very much complementary: that is, the weakness of each is in the area of the strength of the other. It was decided thus to combine the two techniques into one, which we call "robustness analysis."

Experiments with both simulated and real networks have shown that robustness analysis works very well in depicting the strong and the weak points in the network, which have to be judged in three independent senses. The strength/weakness of a network must be studied in the sense of scale, shear, or local twist, each of which provides a different picture of strength. These three indicators (primitives) cannot be combined into a single scalar indicator.

It has been concluded that robustness analysis should be carried out side-by-side with the standard statistical analysis from which it differs fundamentally. It is recommended that the Canadian federal specifications for horizontal geodetic networks be amended to include pertinent prescriptions as far as desired robustness is concerned, i.e., specific robustness to be achieved through meeting robustness tolerance limits.

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1. INTRODUCTION

In Canada, as in most other countries, geodetic networks are designed and classified on the basis of the standard statistical approach. This approach, called in this report simply "covariance analysis," assumes that only random, normally distributed errors are present in the observations. This analysis is oblivious to what may happen to the network if a sizeable error, called here an outlier or a blunder, fails to get intercepted by statistical testing performed during the covariance analysis.

About ten years ago, two groups — one at the University of New Brunswick (UNB) and the other at The University of Calgary (U of C) — independently started a quest for an alternative approach to network design and classification. The U of C group had taken Baarda's [1968] statistically based reliability technique and implemented it for the case of horizontal geodetic networks [Mackenzie, 1985]. It was implemented in a program package called CANDSN [Mepham and Krakiwsky, 1984]. The UNB group took a completely geometrical approach to develop their "geometrical strength analysis " [Dare, 1983] based on using strain as the deformation descriptor. This technique was incorporated in the NETAN program developed for the Geodetic Survey Division of the Canada Centre for Surveying [Craymer et al., 1989].

The idea of looking at the response of geodetic networks to the presence of blunders in observations has been on many people's minds for some time. It was responsible for the Geodetic Survey Division letting out a research contract on 24 August 1989, administered by DSS under SSC file #055SS.23244-9-4198 and #23244-9-4198/01-SS, to UNB with the U of C as a subcontractor. The aims of this contract can be summarized as follows:

(i) to show that the network response to blunders in observations is different than its response to random errors;

(ii) to show the differences between and relative merits of Baarda's reliability analysis and the geometrical strength analysis;

(iii) to demonstrate how the two new techniques work with both a simulated and a real network;

(iv) to present suggestions for updating the Canadian specifications for geodetic horizontal networks to include the reliability/strength aspects.

It became rather obvious at the earliest stages of the investigations that while the reliability analysis is based on rigorous statistical concepts, its treatment of the geometry of virtual (potential) network deformation, which is needed in studying the network response, is rather weak. Conversely, the geometrical strength analysis treats the virtual deformation quite rigorously, while its statistical foundations are weak. Thus, rather than dealing with the two techniques side-by-side, it was decided to combine the advantages of both into one technique called here the "network robustness analysis."

This report, being the final report on the above cited research contract, addresses the required issues in the following way: the three techniques that had to be studied and compared are described in Chapters 2, 3, and 4, respectively. Since the covariance analysis is a rather standard tool, it is presented in a more compact way than the other two techniques. Robustness analysis is discussed in Chapter 5, together with its comparison with geometrical strength and covariance analyses. Chapter 6 is devoted to describing how robustness analysis is implemented on the computer within the framework of the existing NETAN program. The required numerical examples are gathered together in Chapter 7, proposed specifications are in Chapter 8, and our conclusions and recommendations are brought forward in Chapter 9. Suggestions for new federal specifications are submitted as an external appendix to this report.

2. COVARIANCE ANALYSIS

2.1 Introduction

Throughout the discussion in this chapter, we will consider only distance, angle, and azimuth observations. This will simplify our discussion, but some generalizations for the benefit of the reader are thus necessary. We first consider the mathematical model shown below:

$$\mathbf{Ax} = \boldsymbol{\ell} + \mathbf{v},\ \mathbf{C}_{\ell}\quad , \tag{2.1}$$

where
- $\mathbf{A}$ = the design matrix,
- $\mathbf{x}$ = vector of unknown parameters,
- $\boldsymbol{\ell}$ = vector of observations,
- $\mathbf{v}$ = vector of residuals, and
- $\mathbf{C}_{\ell}$ = covariance matrix of the observation.

Equation (2.1) is merely the differential form of a non-linear mathematical model. The equation is formed by linearizing around the Taylor point $\mathbf{x}^{(0)}$ with $\mathbf{x} = \boldsymbol{\delta}$ (correction to initial approximate parameter vector) and $\boldsymbol{\ell} = \mathbf{w}$ (misclosure vector). $\mathbf{x}$ can be solved using the well-known least-squares estimation technique utilizing the normal equations shown below:

$$\mathbf{N}\hat{\mathbf{x}} = \mathbf{A}^t\mathbf{P}_{\ell}\boldsymbol{\ell}\quad , \tag{2.2}$$

where
- $\mathbf{N} = \mathbf{A}^t\mathbf{P}_{\ell}\mathbf{A} + \mathbf{C}_{x^o}^{-1}$ ($\mathbf{C}_{x^o}$ is the a priori covariance matrix for the unknown parameters; it is optional and is not considered further in our discussion), and
- $\mathbf{P}_{\ell} = \sigma_0^2\,\mathbf{C}_{\ell}^{-1}$ (σ_0^2 is the a priori variance factor).

Before we can use the results from our estimation, we need to assess our observations and mathematical model. This allows us to determine if we can rely on the results that we have obtained. The assessment is made using statistical testing. The most important tests that usually are carried out are briefly discussed below.

2.2 Statistical Testing of Observations

Testing of observations is done before they are used to estimate unknown parameters. The reason for testing observations is quality control. We want to know whether the observations that have been collected contain any gross errors. Screening the observations for gross errors before they are used is supposed to ensure that the estimated unknown parameters will not be biased.

The quantities used for testing the observations are either the observations themselves or their residuals. In the latter case, we assume that the observations ℓ are composed of two parts,

$$\ell = \hat{\ell} - \hat{v} \,, \tag{2.3}$$

where $\hat{\ell}$ = the estimated value of the observations,

$\hat{v}$ = the estimated value of the residual of the observations.

Since we are testing only one observation at a time here, univariate testing is used in this context.

There are three types of tests that can be carried out on the observations: namely, :

(a) χ^2 goodness of fit test,

(b) test on the variance, and

(c) test on the mean.

The first test determines whether the histogram of the residuals is compatible with a postulated probability density function (PDF). The PDF that is used here is the normal distribution. This test is important as all the other statistical tests assume that the residuals are normally distributed. (There are, however, statistical tests that do not rely on the normality assumption, known as the non-parametric test [Rao, 1973]. These are seldom used in network analysis and will not be discussed here.)

The second test determines whether the hypothesized population variance σ^2 is compatible with the assessed variance s^2 and can be used only when several values have been collected for one observable. σ^2 can sometimes be viewed as the design variance. If this test fails, then there is a reason to believe that the observations were not collected according to the design. The third test is designed to examine the mean of the data collected for one observable. The comparison between the population mean (μ) and the sample mean ($\bar{l}$) tells us of the presence of possible biases in the observed sample. The last two tests cannot be performed on residuals.

For the three tests described above, there are six situations under which the tests can be carried out. These situations reflect whether the population mean μ and the population variance σ^2 are known or unknown. If μ and/or σ^2 are unknown, they are estimated from the sample mean $\bar{l}$ and the sample variance s^2. It is important to know whether we are treating the population parameters as known or unknown as this will determine the PDF that we should use to carry out the tests. Vaníček and Krakiwsky [1986] explain the tests described above in greater details.

2.3 A Posteriori Testing of Observation and/or Model

The tests use the estimated residuals $\hat{\mathbf{v}}$ or the misclosure vector $\mathbf{w}'$ (when $\mathbf{w}'$ is a function of the observation and not due to the linearization process). The residuals $\hat{\mathbf{v}}$ are indicative of the behaviour of both the observation and the mathematical model. It is generally impossible to separate the two, therefore, the observations and the model are tested simultaneously.

There are two tests that can be conducted on $\hat{\mathbf{v}}$ and $\mathbf{w}'$ depending on whether the variance factor σ_0^2 used for scaling the covariance matrix of observations is known or unknown. When carrying out the test to detect outliers, the covariances between the residuals have to be taken into consideration. In such a situation, the Baarda test statistic should be used or Bonferroni's inequality employed. Both approaches take covariances into account in quite different ways.

The assumption behind the standard testing is that the observations $\boldsymbol{\ell}$ are normally distributed with expected value of $\mathbf{A}\hat{\mathbf{x}}$, i.e.,

$$H_0 : \boldsymbol{\ell} \in n\,(\xi;\, \mathbf{A}\hat{\mathbf{x}},\, \mathbf{C}_{\ell}) \tag{2.4}$$

(Any symmetrical probability distribution function with mean $\mathbf{A}\hat{\mathbf{x}}$ will suffice to satisfy conditions for unbiasness and maximum likelihood of $\hat{\mathbf{x}}$ but not for testing.)

This hypothesis is tested by the "test on the variance factor." The a posteriori variance factor $\hat{\sigma}_0^2$ is computed from

$$\hat{\sigma}_0^2 = \frac{\hat{\mathbf{v}}\ \mathbf{P}_\ell\ \hat{\mathbf{v}}}{\nu} \ , \tag{2.5}$$

where ν is the number of degrees of freedom. If the a priori variance factor σ_0^2 is known, then the null hypothesis for testing is

$$H_0 : \ \frac{\nu\hat{\sigma}_0^2}{\sigma_0^2} \in \chi^2(\xi;\ \nu) \ . \tag{2.6}$$

One of the necessary conditions for this null hypothesis to be satisfied is that the expected value of $\mathbf{v}$ equals to $\mathbf{0}$, i.e., that the observations $\boldsymbol{\ell}$ are burdened only with random errors (with zero-mean). Thus observations are usually again screened against gross errors/biases using tests for outliers.

2.4 Outlier Detection in Observations

Outliers are observations that are considered statistically incompatible with the rest of the series [Vaníček and Krakiwsky, 1986]. This incompatibility is thought to be caused by a blunder made in the measurement or by some sort of disturbance affecting the performance of the measuring system. Outliers can be detected by examining the residuals of the observations after the estimation process.

Because the residuals are mathematically correlated to each other, we would have to work with a multivariate distribution function. This would make the testing procedure quite complex. It is easier and more efficient to work with a univariate distribution. To do this we have to standardize the residuals. Since it is assumed that all the residuals are coming from the same population with different normal density, the standardization process is straightforward. The standardization process is accomplished by the transformation :

$$\tilde{r}_i = \frac{\hat{r}_i}{\sigma_{\hat{r}_i}} \tag{2.7}$$

where $\hat{r}_i = \ell_i - \hat{\ell}_i$ and $\sigma_{\hat{r}_i} = \sigma_{(\ell_i - \hat{\ell}_i)}$.

The univariate tests that are available for outlier detection are shown in Table 13.5 in Vaníček and Krakiwsky [1986]. The table shows the tests when each ℓ_i has been taken out of context, i.e., the question as to whether the other members of the series may also be outliers is deliberately ignored. The tests are thus called the out-of-context tests. An in-context test examines the ℓ_i in light of their existence as one of the members of the series. In this case, the significance level for the test is different from that used in the out-of-context tests. The significance level for the in-context and the out-of-context tests is related by the equation:

$$\alpha \doteq \frac{a}{N} , \tag{2.8}$$

where α = out-of-context significance level,

a = in-context significance level, and

N = the number of observations.

A more detailed description of the out-of-context and the in-context testing can be found in Vaníček and Krakiwsky [1986].

The outlier detection process plays a very important role in our robustness analysis technique as will be shown later. As a matter of fact, there may exist observations in a network that cannot be tested for outliers and the level of detectability varies with network geometry. What happens if ℓ_i burdened with blunders $\Delta\ell_i$, e.g., gross error of bias, are used in the computation? The effect of the blunder and whether it is detectable will depend on the geometry of the network. Figure 2.1 illustrates this point.

Figure 2.1 (a) shows a closed network of points. All the points in the network are determined employing redundant observations. If observation ℓ_3 is burdened with a blunder, it either can be detected from the residual of that observation or has only a small effect on point A as other observations are also used to compute coordinates of the point. In Figure 2.1 (b), the

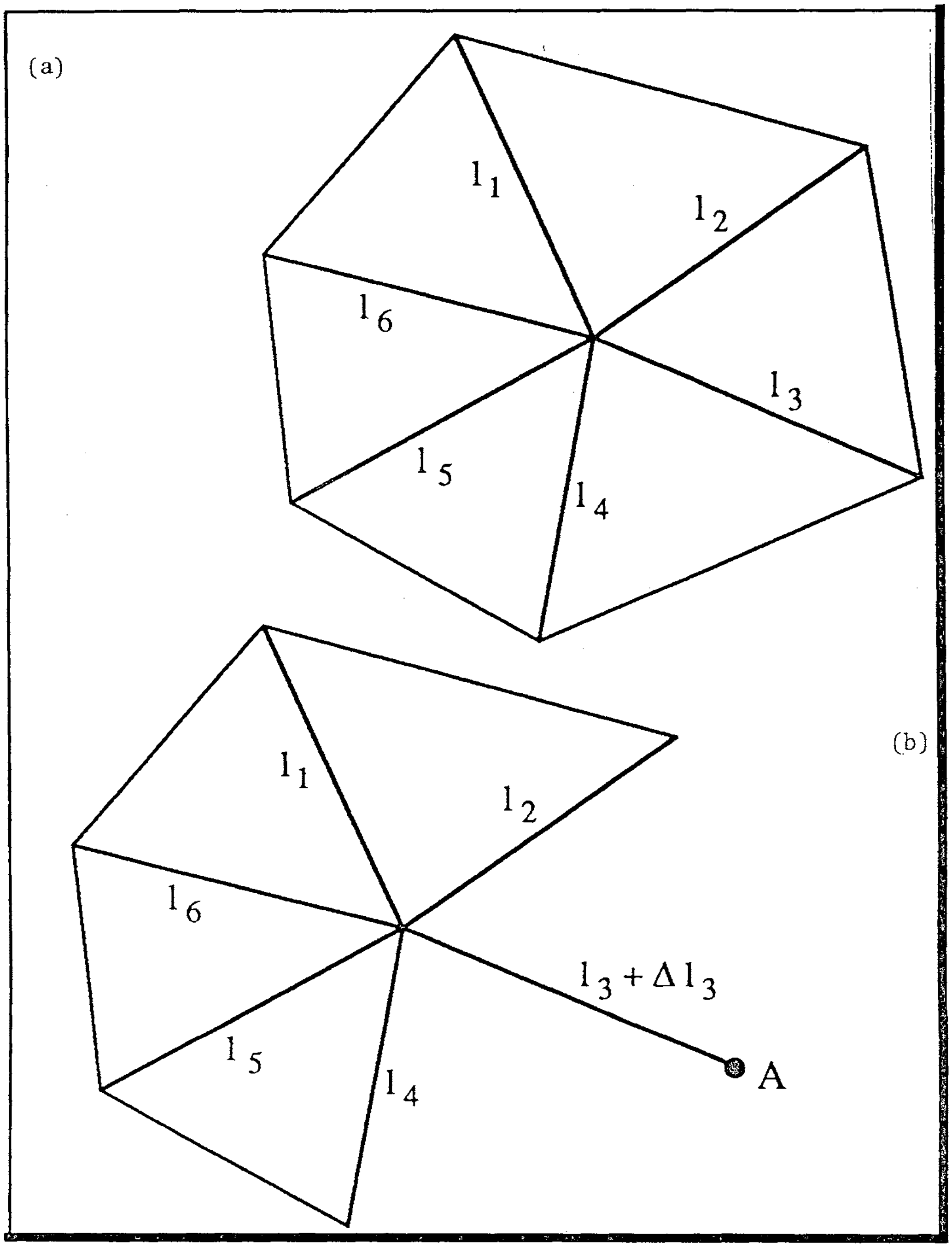

Figure 2.1. Detectability of blunders and their effect on a horizontal network.
(a) A blunder either can be detected or has a small effect on the network.
(b) Blunder $\Delta\ell_3$ cannot be detected and has a large effect on the network (only on point A).

blunder in the observation cannot be detected as there are no redundant measurements made to point A. The effect of the blunder will be large as only the observation burdened with the blunder was used to compute the position of A.

The effect of a blunder on the network depends on the following two circumstances:

(a) if $\Delta\ell_i$ can be detected by statistical testing, and

(b) how the network reacts to the presence of $\Delta\ell_i$.

Both are functions of network geometry, the observation accuracy, and the magnitude of $\Delta\ell_i$. When can a blunder be detected? The answer lies in Baarda's reliability theory explained in Chapter 3.

2.5 Assessment of the Estimated Positions

Once the observations have been screened and the mathematical model examined, the estimated parameters (positions) should be assessed. The assessment consists of the determination of confidence regions (sometimes known as error ellipses for 2D positions or error ellipsoids for 3D positions) for the positions. These represent the amount of trust that one can place on the estimated positions.

The confidence region determined for the estimated position depends on the test statistic y shown below. There are two different cases where y can be determined, i.e., when σ_0^2 is either known or unknown. If σ_0^2 is known then the test statistic used is:

$$y = (x - \hat{x})^t C_{\hat{x}}^{-1} (x - \hat{x}) \quad , \tag{2.9}$$

where x = the unknown parameters (coordinates),

$\hat{x}$ = estimate of the unknown parameters, and

$C_{\hat{x}}$ = covariance matrix of the estimated parameters.

The test statistic y shown above has a χ^2 distribution with u degrees of freedom, where u is also the number of unknown parameters in the estimation process. If σ_0^2 is unknown, then the test y is given by:

$$y = \frac{(x - \hat{x})^t C_{\hat{x}}^{-1} (x - \hat{x})}{u} \tag{2.10}$$

The test statistic shown in equation (2.10) has an $F_{(u,\nu,\infty)}$ distribution (ν = number of observations minus the number of unknown parameters).

For a given significance level α, the critical value of y can be looked up from the χ^2 of the F tables depending on whether σ_0^2 is known or unknown. If we substitute this value for y in equations (2.9) or (2.10), we will get a u-dimensional hyperellipsoid. This hyperellipsoid can be understood as a u-dimensional confidence region centred at $\hat{x}$. Any tested value **x** that falls within the hyperellipsoid must then be considered compatible with $\hat{\mathbf{x}}$ on the level of probability (1-α). Two-dimensional subvectors of **x** similarly fall into 2D confidence regions centred on corresponding 2D subvectors of $\hat{\mathbf{x}}$, i.e., the points of the network. The axes and orientation of the confidence regions can be computed by solving the eigenvalue problem for each confidence ellipse. The equations for computing the axes and orientation of the confidence ellipse can be found in Steeves and Fraser [1983].

There are two types of confidence regions: point confidence and relative confidence region. The point confidence region reflects how accurately the station has been positioned with respect to the 'datum' of the network: point confidence regions thus depend on the datum defining the network. The relative confidence region represents the relative accuracy between the two stations. It is not datum dependent and is most often used to define the accuracy of a network.

The confidence regions are usually computed for a probability level of 39%. Such confidence regions are called the standard confidence regions. This probability level can be increased by multiplying it with an expansion factor. The expansion factor is given by:

$$C_\alpha(u) = \sqrt{\xi_{(y,\ 1\text{-}\alpha)}}. \tag{2.11}$$

where $\xi_{(y,\ 1\text{-}\alpha)}$ is the absence of the appropriate PDF corresponding to the 1-α probability. This expansion factor is multiplied with the axes of the standard confidence region to obtain the 1-α in-context confidence interval.

3. RELIABILITY ANALYSIS

3.1 Baarda's Reliability Theory

Before we go into detail on the theory, the concept of hypothesis testing should be explained first. The role of hypothesis testing is to allow us to make a statistical decision concerning postulated population parameters, e.g., mean μ or variance σ^2, etc., to have some particular value. This is called the null hypothesis (H_0). For every null hypothesis, there exists an infinite number of alternative hypotheses (H_1), each of which states that the population parameters have some other particular values.

When we perform hypothesis testing, there are only two possible outcomes, i.e., to accept H_0 or to reject H_0. Similarly, there are two possible outcomes for the test of the alternative hypothesis H_1. None of the hypotheses may be true, in which case the test at least should tell us which hypothesis is better. To make a definite decision concerning H_0, we need to have an infinite sample to work with. Since this is never available, a decision made on a finite sample should be trusted only to a certain degree. Such a decision has attached to it only a limited confidence.

The probability α of rejecting H_0 when in fact H_0 is true (Type I error) is called the significance level. The complementary probability ($1-\alpha$) is called the confidence level, and it is the measure of confidence we have in the decision (Type I error). Likewise, a situation might arise that H_0 is false and we accept it. This is called the Type II error. The probability of making this decision is β. ($1-\beta$) is called the power of the test, and it expresses the confidence we have in the decision made. The situation described above can be summarized in Table 3.1.

When Baarda first developed his reliability theory, he treated the blunders as unknown parameters to be estimated, i.e., the blunders are treated as deterministic quantities. Most of the research work and literature dealing with outliers or blunders treat them as deterministic quantities that have to be estimated.

Table 3.1. Testing of a null hypothesis H_0 against an alternative hypothesis H_1 (after Vaníček and Krakiwsky [1986]).

Situation \ Decision	Test tells us to accept H_0	Test tells us to reject H_0
H_0 true	Correct decision Probability = $1-\alpha$ (confidence level)	Type I error Probability = α (significance level)
H_0 false (H_1 true)	Type II error Probability = β	Correct decision Probability = $1-\beta$ (power of test)

The blunders can be estimated from the residuals obtained after a least-squares adjustment. The relationship between the observational errors and the residuals is shown below [Stefanovic, 1978; Kavouras, 1982]:

$$\hat{v} = Q_{\hat{v}} C_{\ell}^{-1} \varepsilon \quad , \tag{3.1}$$

where $\hat{v}$ = the estimated residuals,

ε = the true observational errors,

$Q_{\hat{v}}$ = the cofactor matrix of the estimated residuals, and

C_{ℓ} = the covariance matrix of the observations.

The cofactor matrix and the covariance matrix of the residuals is related by σ_0^2 [Mikhail, 1976]. Therefore, if σ_0^2 is assumed known, then equation (3.1) can be rewritten as:

$$\hat{v} = C_{\hat{v}} C_{\ell}^{-1} \varepsilon \quad , \tag{3.2}$$

where $C_{\hat{v}}$ is the covariance matrix of the residuals.

3.2 Effect of Blunders

Now, if we assume ε to be made up of two parts consisting of a random part ε_r and a gross error part (blunder) $\nabla\ell$, we have:

$$\varepsilon = \varepsilon_r + \nabla\ell \quad , \tag{3.3}$$

Substituting equation (3.3) into equation (3.2), we have:

$$
\begin{aligned}
\hat{v} &= C_{\hat{v}} C_{\ell}^{-1} (\varepsilon_v + \nabla \ell) \\
&= C_{\hat{v}} C_{\ell}^{-1} \varepsilon_r + C_{\hat{v}} C_{\ell}^{-1} \nabla \ell \\
\hat{v} &= v_r + \nabla v \ ,
\end{aligned} \tag{3.4}
$$

where v_r = influence of the random error on the residual,

∇v = influence of the blunder on the residual.

If an observation ℓ_i is not burdened with a blunder, then $\nabla\ell_i$ will be zero. On the other hand, if observation ℓ_i is burdened with a blunder, then $\nabla\ell_i$ will be non-zero. Therefore, through testing of the residuals $\hat{v}$, an observation containing a blunder could be detected.

In carrying out the statistical test on our observations, we always assume that the observations are normally distributed with mean μ and variance σ^2. If an observation is burdened with a blunder, then it will have a distribution with mean, say, $\mu + \sqrt{\lambda}$, and variance of σ^2. This method of modelling the blunder is called the mean shift model [Chen et al., 1987] where the (unknown) mean shift is given by $\sqrt{\lambda}$. The PDF of the observation containing a blunder is shifted by $\sqrt{\lambda}$ from its own PDF not burdened by a blunder. This situation is depicted in Figure 3.1.

One of the statistical tests carried out after a least-squares adjustment is to test the estimated reference variance $\hat{\sigma}_0^2$ against a hypothesized reference variance σ_0^2, as described in the previous chapter.

There are many reasons why the test can fail and H_0 be rejected. Some of these reasons can be found in Uotila [1976], or Vaníček and Krakiwsky [1986]. For our purpose, however, we shall assume that the reason why the test fails is that blunders exist in our observations. This is a valid assumption because blunders have an influence on $\hat{v}$. Since $\hat{\sigma}_0^2$ is estimated using $\hat{v}$, the presence of blunders will cause the distribution of test statistic y to be shifted by λ. The shift λ is also known as the non-centrality parameter [Mackenzie, 1985]. The amount of shift can be computed from the blunders themselves, through their influence ∇v. The derivation of λ using ∇v is discussed below.

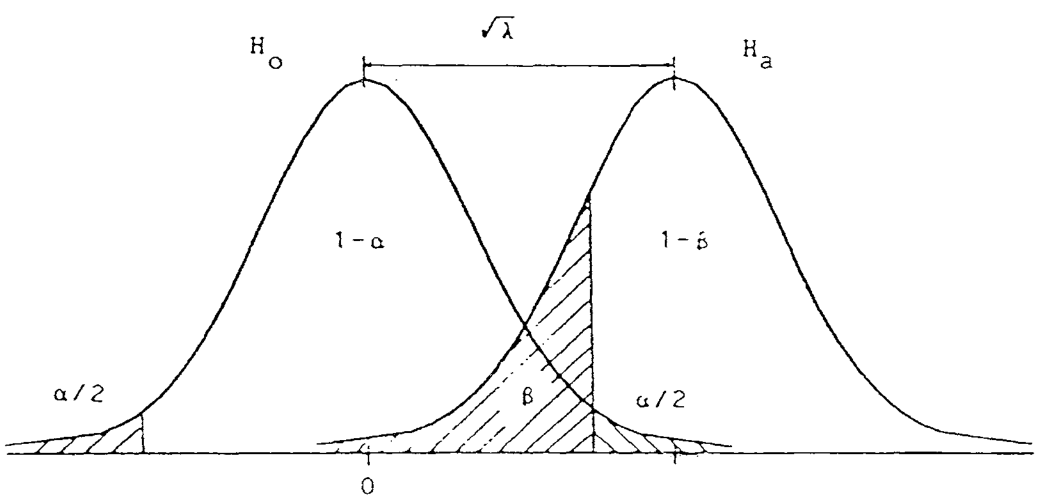

Figure 3.1 The central and non-central normal distribution (after Mackenzie [1985]).

3.3 Formulation of an Alternative Hypothesis

Under the null hypothesis, the expectation of $y' = y/\nu$ is 1, as seen from equation (2.6). In an alternative hypothesis, the expected value of y' is not equal to 1. We can write the expected value of y' under H_1 in two parts as shown below:

$$E[y'|H_1] = E[y'|H_0] + \nabla[y'] \tag{3.5}$$

where $\nabla y'$ is the amount by which the χ^2 distribution has shifted due to the presence of blunders. Therefore,

$$E[y'|H_1] = 1 + \nabla[y'] \ . \tag{3.6}$$

But $y' = \hat{\sigma}_0^2 / \sigma_0^2$ and substituting y into equation (3.6) we get:

$$\nabla[y'] = \nabla\,(\hat{\sigma}_0^2/\sigma_0^2)$$

$$= \nabla\hat{\sigma}_0^2/\sigma_0^2$$

where $\nabla\hat{\sigma}_0^2$ is the amount of shift of $\hat{\sigma}_0^2$ due to the presence of blunders

$$\nabla[y'] = \frac{1}{\sigma_0^2}(\nabla v^t\, C_\ell^{-1}\, \nabla v)/\nu \ . \tag{3.7}$$

Equation (3.6) can be rewritten as:

$$E[y'|H_1] = 1 + \frac{1}{\sigma_0^2}\,(\nabla v^t\, C_\ell^{-1}\, \nabla v)/\nu$$

$$= 1 + \lambda/\nu$$

where

$$\lambda = \frac{1}{\sigma_0^2}\ \nabla v^t\, C_\ell^{-1}\, \nabla v \ . \tag{3.8}$$

Since $\nabla v = C_{\hat{v}}\, C_\ell^{-1}\, \nabla\ell$ (equation (3.4)), we can write equation (3.8) as

$$\lambda = \frac{1}{\sigma_0^2}\,(C_{\hat{v}}\, C_\ell^{-1}\, \nabla\ell)^t\, C_\ell^{-1}\, C_{\hat{v}}\, C_\ell^{-1}\, \nabla\ell \tag{3.9a}$$

$$= \frac{1}{\sigma_0^2}\ \nabla\ell^t\, C_\ell^{-1}\, C_{\hat{v}}\, C_\ell^{-1}\, C_{\hat{v}}\, C_\ell^{-1}\, \nabla\ell \ . \tag{3.9b}$$

Using the indempotence property of $C_{\hat{v}}\, C_\ell^{-1}$ [Mikhail, 1976], we can write,

$$\lambda = \frac{1}{\sigma_0^2} \nabla\ell^t C_\ell^{-1} C_{\hat{v}} C_\ell^{-1} \nabla\ell \quad . \tag{3.9c}$$

If $\sigma_0^2 = 1$, then equation (3.9c) is reduced to:

$$\lambda = \nabla\ell^t C_\ell^{-1} C_{\hat{v}} C_\ell^{-1} \nabla\ell \quad . \tag{3.9d}$$

Figure 3.2 shows the shift of the probability distribution function of H_0 due to blunders $\nabla\ell$ in the observations.

When formulating the reliability technique, we are not interested in the magnitude of $\nabla\ell$ itself. What is important is to know the magnitude of the blunder that cannot be detected. To be able to determine this we need to know λ. Since $\nabla\ell$ is unknown, however, λ cannot be computed using equation (3.9d). Instead, we can select a critical value λ_0 (based upon selected α_0 and β_0, as shown later) to determine the magnitude of $\nabla\ell$ that cannot be detected. Equation (3.9d) can then be rewritten as

$$\lambda_0 = \nabla_0\ell^t C_\ell^{-1} C_{\hat{v}} C_\ell^{-1} \nabla_0\ell \quad . \tag{3.10}$$

In carrying out the test for detecting blunders in our observations, we have assumed that only one blunder at a time was present: each observation is tested in turn to see if it is burdened with a blunder. The hypothesis set up for each observation is

$$\forall i = 1, n \quad : \quad \begin{cases} H_{0i} : \nabla\ell_i = 0 \\ H_{1i} : \nabla\ell_i \neq 0 \end{cases} \quad .$$

Baarda called the consecutive testing of the alternative hypotheses H_{1i} "a data snooping strategy" [Baarda, 1968; Kok, 1984].

3.4 Redundancy Measure

Since we are testing one observation at a time, the test hypothesis will be one dimensional [Kok, 1984]. Therefore, we only have to deal with a univariate probability distribution. Baarda [1968] has ascertained that in testing the residuals to detect the presence of blunders,

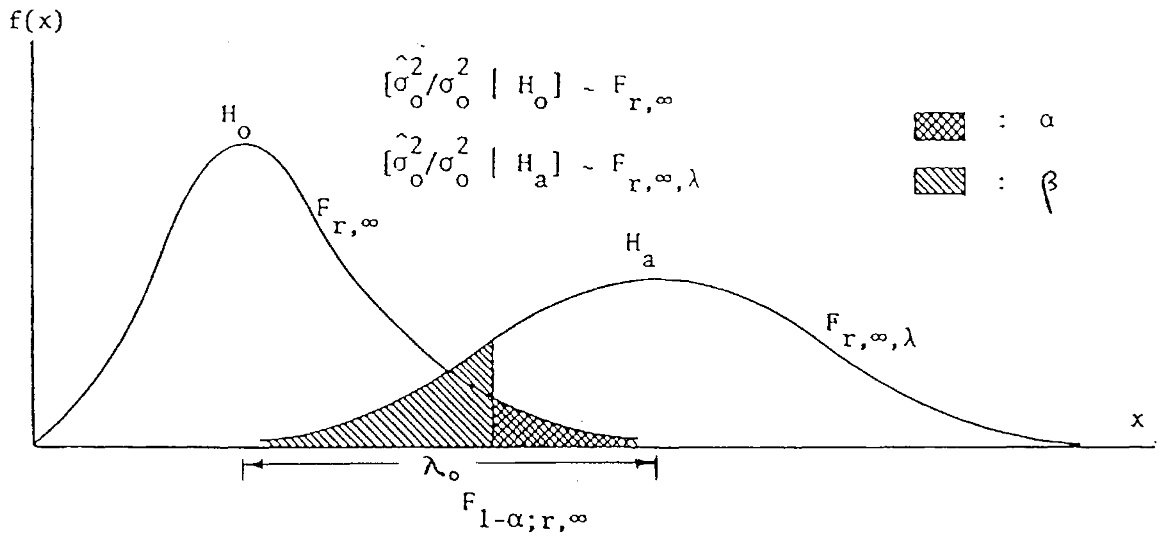

Figure 3.2 Probability distribution function of test statistic y under H_0 and H_1 (after Kavouras [1982]).

the most sensitive test quantity is the weighted residual. If we test one observation at a time, the weighted residual of observation ℓ_i is given by

$$\forall i \ : \ v_i^* = (C_\ell^{-1} \hat{v})_i \ . \tag{3.11}$$

The variance of the weighted residual in equation (3.11) is given by:

$$\forall i \ : \ (\sigma_{vi}^*)^2 = (C_\ell^{-1} C_{\hat{v}} C_\ell^{-1})_{ii} \ . \tag{3.12}$$

The test statistic that is used to test the hypothesis above is the standardized residual having a standard normal distribution with $\mu = 0$ and $\sigma^2 = 1$. The test statistic is thus

$$\forall i \ : \ w_i = \frac{v_i^*}{\sigma_{vi}^*} \ . \tag{3.13}$$

If $|w_i| > n_{(1-\alpha)}$ then H_{0i} is rejected. If H_{1i} is true, then observation ℓ_i is burdened with a blunder. Under $H_{1i,}$ w_i will have a non-central standard normal distribution. The amount of shift with respect to the central distribution is given by $\sqrt{\lambda_0}$. This situation is depicted in Figure 3.3. Under the null hypothesis, the expectation of w_i is zero

$$E[w_i|H_0] = 0 \ , \tag{3.14}$$

and under the alternative hypothesis, the expectation of w_i is $\sqrt{\lambda_0}$

$$E[w_i|H_1] = \sqrt{\lambda_0} \ . \tag{3.15}$$

The practical application of Baarda's reliability theory is to determine the magnitude of blunders that cannot be detected on a given probability level α_0 when accepting a level of risk β_0 of committing a Type II error (accepting that there is no blunder present when there is one present). If we assume that all our observations are burdened with blunders, then we are interested in the minimum size of blunder in each observation that can still be detected. Therefore, having preselected α_0 and β_0 (the selection of these two quantities will be discussed later), $\sqrt{\lambda_0}$ can be determined. Figure 3.3 shows how $\sqrt{\lambda_0}$ can be computed using the formula shown below:

$$\sqrt{\lambda_0} = \xi_{n(0,1),1-\alpha/2} + \xi_{n(0,1),1-\beta} \ . \tag{3.16}$$

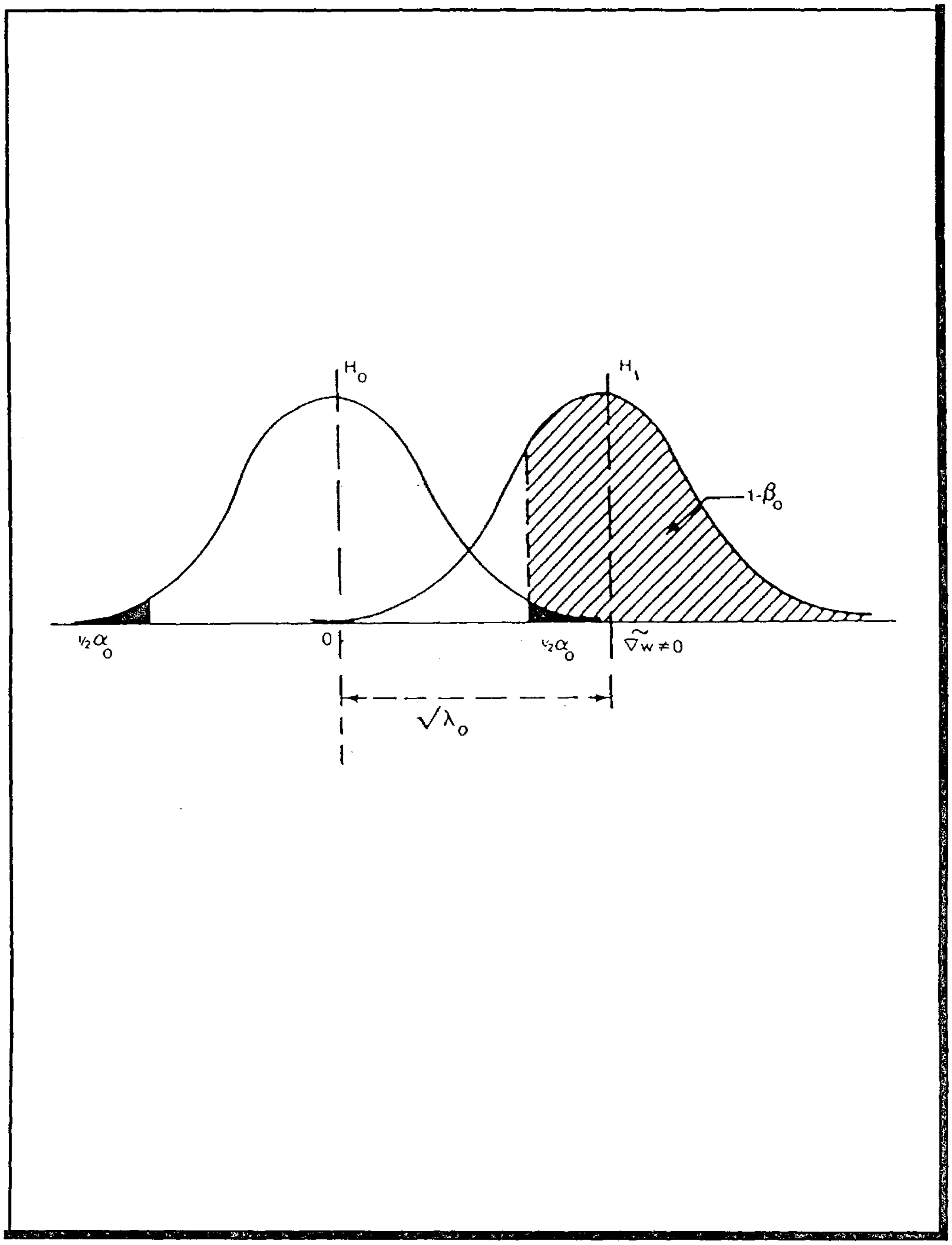

Figure 3.3 $\sqrt{\lambda_0}$ shows the shift of the standard normal distribution of w when H_1 is true (after Kok [1984]).

The maximum undetectable blunder (or minimum detectable blunder) can be computed using equation (3.10). Since we are testing one observation ℓ_i at a time, equation (3.10) can be rewritten as:

$$\forall i \;:\; \lambda_0 = \nabla_0 \ell_i^2 \,(C_\ell^{-1})_{ii}\,(C_{\hat{v}})(C_\ell^{-1})_{ii} \;. \tag{3.17}$$

In equation (3.17), the product $(C_{\hat{v}})(C_\ell^{-1})_{ii}$ is called the redundancy number r_i [Forstner, 1979; Mackenzie, 1985]. The r_i gives an insight into the 'controllability' of the observations. An observation is said to be 'fully controlled' if all of the observational errors (random and non-random) show up in the estimated residual [Mackenzie, 1985]. This is obvious from equation (3.2) which can be rewritten as

$$\hat{\mathbf{v}} = \mathbf{r}\,\boldsymbol{\varepsilon} \tag{3.18}$$

where $\mathbf{r} = C_{\hat{v}}\, C_\ell^{-1}$. Rewriting now equation (3.17), we get

$$\forall i \;:\; \lambda_0 = \nabla_0 \ell_i^2 \,(C_\ell^{-1})_{ii}\, r_i \;. \tag{3.19}$$

Rearranging equation (3.19), we arrive at

$$\forall i \;:\; \nabla_0 \ell_i^2 = \frac{\lambda_0}{r_i\,(C_\ell^{-1})_{ii}}$$

or

$$\forall i \;:\; \nabla_0 \ell_i = \frac{\sqrt{\lambda_0}}{\sqrt{r_i}\,\sqrt{(C_\ell^{-1})_{ii}}} \;. \tag{3.20a}$$

Since $(C_\ell)_{ii} = \sigma_{\ell i}^2$, we have

$$\forall i \;:\; \nabla_0 \ell_i = \sigma_{\ell i}\,\frac{\sqrt{\lambda_0}}{\sqrt{r_i}} \;. \tag{3.20b}$$

$\nabla_0 \ell_i$ is called the internal reliability measure [Baarda, 1968]. It represents the maximum blunder in an observation undetectable with selected α_0 and β_0. We note that if $r_i = 0$, no error can be detected by the outlier test (case requiring only the minimal number of observations linking the point to the rest of the network). The $r_i = 1$ case represents the other extreme when any $\sqrt{\lambda_0}$ multiple of σ_{ℓ_i} could be recognized as an outlier.

3.5 External Reliability

Another reliability measure that Baarda developed in 1968 is called the external reliability. External reliability tells us about the effect of $\nabla_0 \ell_i$ on the positions obtained through the least-squares adjustment. External reliability, however, is not used in our method; the effect of $\nabla_0 \ell_i$ on the result is handled in a different way. To make our discussion more complete and to make a comparison between our method and Baarda's method in assessing the effect of $\nabla_0 \ell_i$, a brief discussion on the external reliability is still given below.

In the least-squares adjustment, the unknown parameters are estimated using equation (1.2). If we pre-multiply equation (1.2) by $\mathbf{N}^{-1}$, we get:

$$\hat{\mathbf{x}} = \mathbf{N}^{-1}\,\mathbf{A}^t\,\mathbf{P}_\ell\,\boldsymbol{\ell}\ , \tag{3.21a}$$

or

$$\hat{\mathbf{x}} = (\mathbf{A}^t\,\mathbf{P}_\ell\,\mathbf{A})^{-1}\,\mathbf{A}^t\,\mathbf{P}_\ell\,\boldsymbol{\ell}\ . \tag{3.21b}$$

Supposing that the observation vector $\boldsymbol{\ell}$ is burdened by blunders $\nabla\boldsymbol{\ell}$, i.e., $\boldsymbol{\ell}' = \boldsymbol{\ell} + \nabla\boldsymbol{\ell}$ and substituting $\boldsymbol{\ell}'$ into equation (3.21b), we get

$$\hat{\mathbf{x}}' = (\mathbf{A}^t\,\mathbf{P}_\ell\,\mathbf{A})^{-1}\,\mathbf{A}^t\,\mathbf{P}_\ell\,\boldsymbol{\ell}'\ , \tag{3.22}$$

where $\hat{\mathbf{x}}'$ are the shifted unknown parameters affected by blunders $\nabla\boldsymbol{\ell}$. Substituting for $\boldsymbol{\ell}'$ into equation (3.22), we get

$$\begin{aligned}\hat{\mathbf{x}}' &= (\mathbf{A}^t\,\mathbf{P}_\ell\,\mathbf{A})^{-1}\,\mathbf{A}^t\,\mathbf{P}_\ell\,(\boldsymbol{\ell} + \nabla\boldsymbol{\ell})\\ &= (\mathbf{A}^t\,\mathbf{P}_\ell\,\mathbf{A})^{-1}\,\mathbf{A}^t\,\mathbf{P}_\ell\,\boldsymbol{\ell} + (\mathbf{A}^t\,\mathbf{P}_\ell\,\mathbf{A})^{-1}\,\mathbf{A}^t\,\mathbf{P}_\ell\,\nabla\boldsymbol{\ell}\ .\end{aligned} \tag{3.23}$$

and denoting by $\nabla\hat{\mathbf{x}}$ the shift of $\hat{\mathbf{x}}$ due to $\nabla\boldsymbol{\ell}$ yields

$$\nabla\hat{\mathbf{x}} = (\mathbf{A}^t\,\mathbf{P}_\ell\,\mathbf{A})^{-1}\,\mathbf{A}^t\,\mathbf{P}_\ell\,\nabla\boldsymbol{\ell}\ . \tag{3.24}$$

The effect of the maximum undetectable blunder $\nabla_0 \ell_i$ on the estimated parameters can be determined by substituting $\nabla_0 \ell_i$ for $\nabla\boldsymbol{\ell}$:

$$\forall i\ :\ \nabla_0 \hat{x}_i = (\mathbf{A}^t\,\mathbf{P}_\ell\,\mathbf{A})^{-1}\,\mathbf{A}^t\,\mathbf{P}_\ell\ \nabla_0 \ell_i\ . \tag{3.25}$$

Here, $\nabla_0 \hat{x}_i$ is dependent on the coordinate definition, i.e., it is datum dependent. Baarda [1976; 1979] proposed another kind of external reliability measure:

$$\lambda_{0i} = (\nabla_0 \hat{x}_i)^t \, C_{\hat{x}}^{-1} \, (\nabla_0 \hat{x}_i) \quad . \tag{3.26}$$

known as the relative external reliability measure.

As we shall see in the next chapter, the effect of blunders on the network is better handled as a virtual deformation and thus depicted by a more appropriate technique than the 'external reliability.'

4. GEOMETRICAL STRENGTH ANALYSIS

In this chapter, the use of strain for the strength analysis of geodetic networks is described. The basic concepts of strain are given followed by its application to geodetic networks, and specifically its use as a tool for analysing the geometrical strength of networks. We also show that changes in the network datum have only a second-order effect on strength.

4.1 Concept of Strain

Strain is a purely geometric approach to the analysis of the deformation of a physical body. It is based on the theory of elasticity in mechanics where it is applied to the description of the relative deformation of a body with respect to some initial state. Here deformation is taken to mean the change in shape or configuration of the body.

Deformation can be classified as either homogeneous or nonhomogeneous. If the deformation is homogeneous, straight or parallel lines will remain as straight or parallel lines after deformation. If, on the other hand, the deformation is nonhomogeneous, initially straight or parallel lines become curved or nonparallel after deformation. These deformations are illustrated in Figure 4.1.

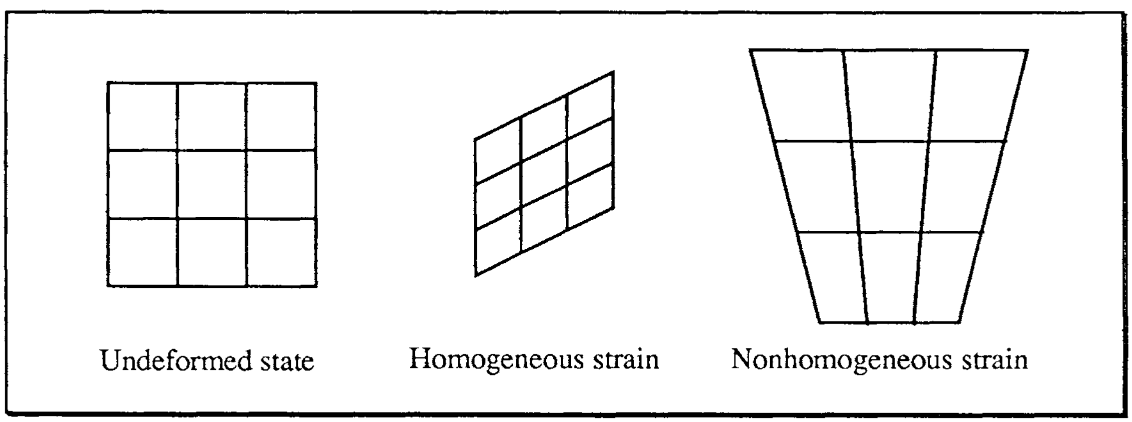

Figure 4.1 Examples of homogeneous and nonhomogeneous deformations.

Strain induced by homogeneous deformation is called homogeneous strain and is constant over all parts of the region of deformation. Nonhomogeneous deformation, on the other hand, produces a more complicated nonhomogeneous strain field.

Deformation and strain can also be classified as finite or infinitesimal. Finite strain usually describes an instantaneous deformation of a continually deforming body with respect to its original undeformed state, i.e., cumulative strain. On the other hand, infinitesimal or incremental strain describes the instantaneous deformation of the current deformed state with respect to some earlier, not necessarily undeformed, instantaneous state.

Only nonhomogeneous and infinitesimal deformation is needed in strength analysis due to the following considerations:

(a) the deformation of a geodetic network is generally nonhomogeneous, and

(b) in the strength analysis of a geodetic network, the deformation is much smaller compared to the size of the network and we can thus use infinitesimal strain theory.

The latter allows us to take advantage of the fact that infinitesimal deformation is differentially small in order to simplify the mathematical description of strain.

Mathematically, infinitesimal strain is defined as the rate of change (i.e., gradient or slope) of an object's displacement field with respect to position. Given a three-dimensional (3D) displacement field $\mathbf{u}(x,y,z)=(u,v,w)^T$, as a function of position $\mathbf{x}=(x,y,z)^T$, the strain matrix E consists of 9 linear displacement gradients given by

$$\mathbf{E} = \text{grad}(\mathbf{u}) = \frac{\partial \mathbf{u}(x,y,z)}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix} = \begin{bmatrix} e_{ux} & e_{uy} & e_{uz} \\ e_{vx} & e_{vy} & e_{vz} \\ e_{wx} & e_{wy} & e_{wz} \end{bmatrix}, \qquad (4.1)$$

where the derivatives are evaluated at the point of concern. These linear strains e correspond to the rate of change of displacement in each of the three coordinate components along the three coordinate axes. For example, e_{uy} is the rate of change or gradient of displacement in the x-direction with respect to position in the x-direction.

Note that the mechanical properties of the material are not involved in strain. Strain is applicable, whatever the mechanical behaviour of the material. Note also that strains describe only the relative displacement of points so that rigid body translations do not affect strain. This will be discussed in more detail later in this chapter.

4.2 Deformation Primitives

The strain matrix contains all of the strain information about the displacement field. It is not easily interpreted, however. Various scalar parameters can be derived from the strain matrix in order to make the interpretation of strain more convenient and illustrative. We call these parameters deformation primitives.

The strain matrix **E** can be decomposed into its symmetric **S** and anti-symmetric **A** parts; i.e.,

$$\mathbf{E} = \mathbf{S} + \mathbf{A} \quad , \tag{4.2}$$

where

$$\mathbf{S} = \begin{bmatrix} \varepsilon_{ux} & \varepsilon_{uy} & \varepsilon_{uz} \\ \varepsilon_{vx} & \varepsilon_{vy} & \varepsilon_{vz} \\ \varepsilon_{wx} & \varepsilon_{wy} & \varepsilon_{wz} \end{bmatrix} = \begin{bmatrix} e_{ux} & \frac{1}{2}(e_{uy}+e_{vx}) & \frac{1}{2}(e_{uz}+e_{wx}) \\ \frac{1}{2}(e_{uy}+e_{vx}) & e_{vy} & \frac{1}{2}(e_{vz}+e_{wy}) \\ \frac{1}{2}(e_{uz}+e_{wx}) & \frac{1}{2}(e_{vz}+e_{wy}) & e_{wz} \end{bmatrix} , \tag{4.3}$$

$$\mathbf{A} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2}(e_{uy}-e_{vx}) & \frac{1}{2}(e_{uz}-e_{wx}) \\ \frac{1}{2}(e_{uy}-e_{vx}) & 0 & -\frac{1}{2}(e_{vz}-e_{wy}) \\ -\frac{1}{2}(e_{uz}-e_{wx}) & \frac{1}{2}(e_{vz}-e_{wy}) & 0 \end{bmatrix} . \tag{4.4}$$

The symmetric part is often referred to as the symmetric strain tensor.

The symmetric strain tensor **S** describes the expansion and contraction as well as the shearing deformation at a point. The strain tensor is usually parameterized in terms of the so-called strain ellipse or ellipsoid in the same manner that error ellipses and ellipsoids are computed from covariance matrices, except that no square roots of the semi-axis lengths are taken. The principal strains (λ_1, λ_2, λ_3) are the eigenvalues of the strain tensor and the eigenvectors are the directions of the principal axes. Negative principal strains indicate contraction and positive principal strains expansion.

The anti-symmetric strain matrix **A** describes the twisting deformation at a point. The quantities ω are called average differential rotations and describe the twisting about each of the three coordinate axes at a point. Note that in the two-dimensional case there is only a twist ω_z about the local z-axis (i.e., in the x-y horizontal plane).

More convenient scalar deformation primitives for strength analysis can also be derived from the strain matrix (see Schneider [1982]). Dilation σ describes the average extension or contraction at a point and is defined as the average of the principal strains; e.g., for 3D

$$\sigma = \frac{\lambda_1+\lambda_2+\lambda_3}{3} = \frac{\varepsilon_{ux}+\varepsilon_{vy}+\varepsilon_{wz}}{3} = \frac{e_{ux}+e_{vy}+e_{wz}}{3}\,. \tag{4.5}$$

Note that the sum of principal strains is equivalent to the trace of the symmetric strain tensor which is equal to the trace of the strain matrix. Total strain λ is a similar quantity, defined as the geometric mean of the principal strains [Dare, 1983]; i.e.,

$$\lambda = \sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}\,. \tag{4.6}$$

Shear strain can be classified as either pure shear or simple shear. Pure shear τ deforms a square into a rectangle so that separation between lines changes. It is defined by [Schneider, 1982]

$$\tau_{xy} = -\tau_{yx} = \frac{1}{2}(e_{ux} - e_{vy}) = \frac{1}{2}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right), \tag{4.7}$$

$$\tau_{xz} = -\tau_{zx} = \frac{1}{2}(e_{ux} - e_{wz}) = \frac{1}{2}\left(\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}\right), \tag{4.8}$$

$$\tau_{yz} = -\tau_{zy} = \frac{1}{2}(e_{vz} - e_{wy}) = \frac{1}{2}\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right). \tag{4.9}$$

Simple shear υ deforms a rectangle into a rhombus so that angles between lines change. It is defined as [Schneider, 1982]

$$\upsilon_{xy} = -\upsilon_{yx} = \frac{1}{2}(e_{uy} + e_{vx}) = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{4.10}$$

$$\upsilon_{xz} = -\upsilon_{zx} = \frac{1}{2}(e_{uz} + e_{wx}) = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \tag{4.11}$$

$$\upsilon_{zz} = -\upsilon_{zy} = \frac{1}{2}(e_{vy} + e_{wz}) = \frac{1}{2}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right). \tag{4.12}$$

Neither type of shear produces any rotation. These two types of shear are illustrated in Figure 4.2. Another type of shear, total shear γ, is the geometric mean of the components of pure and simple shear; i.e.,

$$\gamma_{xy} = \sqrt{\tau_{xy}^2 + \upsilon_{xy}^2} , \tag{4.13}$$

$$\gamma_{xz} = \sqrt{\tau_{xz}^2 + \upsilon_{xz}^2} , \tag{4.14}$$

$$\gamma_{yz} = \sqrt{\tau_{yz}^2 + \upsilon_{yz}^2} . \tag{4.15}$$

The principal axes of the strain tensor define the directions in which no shear takes place. The directions of maximum shear are at 45° to the principal axes of the strain ellipse/ellipsoid. The magnitude of shear can also be determined indirectly from the difference of the principal strains (lengths of the principal axes of the strain ellipse/ellipsoid) [Schneider, 1982].

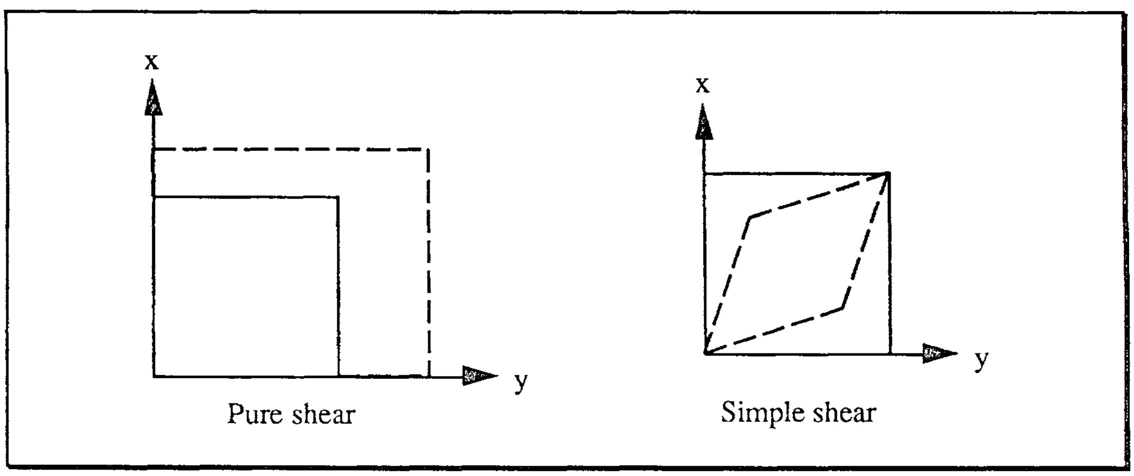

Figure 4.2 Pure and simple shear. Solid lines are the undeformed state and broken lines are the deformed state.

Note that the symmetric strain tensor S can be represented in terms of dilation, pure shear, and simple shear. Using the above definitions, we find that

$$\mathbf{S} = \begin{bmatrix} \sigma+\tau_{xy}+\tau_{xz} & \upsilon_{xy} & \upsilon_{xz} \\ \upsilon_{yx} & \sigma+\tau_{xy}+\tau_{xz} & \upsilon_{yz} \\ \upsilon_{zx} & \upsilon_{zy} & \sigma+\tau_{xy}+\tau_{xz} \end{bmatrix} . \tag{4.16}$$

Although the expressions for the various deformation primitives have been developed in 3D, previous investigations by Craymer et al. [1987] have found that only 2D primitives have any practical meaning in the context of geodetic networks. The problem is that geodetic networks are inherently only 2D in nature since they lie on the surface of the Earth whose variations in height are much smaller than those in the horizontal dimension. When two points have very nearly the same height (a common occurrence), the displacement field gradients with respect to height can become extremely large or even discontinuous, resulting in artificially large and misleading results.

The deformation can be displayed in a variety of ways (see Thapa [1980], Schneider [1982], Dare [1983], and Craymer [1987]). In network strength applications, the only scalar primitives needed are differential rotation, dilation, and total shear. These scalar deformation primitives are most conveniently displayed using either 3D surfaces or contour plots. Only the latter is currently supported in the NETAN software which implements this analysis.

4.3 Virtual Deformation of Geodetic Networks

The concept of strain can be readily applied to the analysis of geodetic networks by considering the network to be a structure in itself. That is, stations are held together by the interconnecting observations as a building is held together by its beams. In this analogy, stations are considered to be the joints and observations are the beams and brackets. Distance observations can be thought of as beams of rigid length whose orientation in space is not fixed. Angles can be considered as brackets which fix the relative orientation (angles) between beams of arbitrary length. Azimuths can be thought of as brackets that fix the orientation of a beam of arbitrary length with respect to the foundation (which acts as the datum definition). We have found that using such an analogy helps in the interpretation of the strain parameters.

In practice, the displacement field over a structure is never known as a continuous function of the position of points on the body. The displacements are known only for a discrete set of points describing the structure. Only a discrete displacement field can therefore be obtained which approximates the actual continuous displacement field.

For the strain analysis of geodetic networks, the displacements are of a virtual nature. They represent changes to the coordinates of the points in the network that may result from a variety of changes (perturbations) of the network. Some of these are:

- changes in observation values,
- changes in observation weights,
- deletion or addition of observations,
- deletion or addition of points,
- changes in network constraints.

The virtual displacement field is the set of coordinate changes for all points in the network. Only virtual displacements due to changes of observation values are needed in strength analysis.

Given a local displacement field δ around a point, the strain can be easily determined from the displacement gradient evaluated at the point. For geodetic networks, we can define the "local displacement field" at a point to consist of displacements of either all interconnected points (i.e., all points connected by observations to the point of interest) or all stations within a specified radius of the point of interest. The virtual displacements (i.e., changes in coordinates) of all points within the local displacement field can then be approximated by a simple surface such as a plane or low-degree algebraic surface (surface described by a low-degree algebraic polynomial). In our experience, we have found a plane to be the most robust approximation of the local displacement field at each point. Higher-order algebraic polynomials are not suitable for such applications since they tend to produce spurious gradients when the points are not regularly distributed in space (i.e., they tend to 'fall through' areas without stations).

The gradients of the local displacement field are evaluated separately for each of the coordinate components. A separate local displacement field is determined for each coordinate component and the gradients along each of the coordinate axes are evaluated to give the components of the strain matrix. Fitting a plane surface to each displacement field results in a very simple determination of the strain; the strain components are just the slopes of the planes along each of the coordinate axes.

For the 2D case, the local displacement field components **u** and **v** are approximated by

$$\mathbf{u} = a_0 + a_1\,\mathbf{x} + a_2\,\mathbf{y}\,, \tag{4.17}$$

$$\mathbf{v} = b_0 + b_1\,\mathbf{x} + b_2\,\mathbf{y}\,, \tag{4.18}$$

where **x** and **y** are the coordinate components of the points in the local displacement field, and the a's and b's are the coefficients defining the planes. For numerical stability, these coordinates are expressed relative to the point of interest. Solving for the coefficients in both sets of equations results in

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \mathbf{N}^{-1}\,\mathbf{A}^T\mathbf{u}\,, \tag{4.19}$$

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \mathbf{N}^{-1}\,\mathbf{A}^T\mathbf{v}\,, \tag{4.20}$$

where $\mathbf{N}=\mathbf{A}^T\mathbf{A}$ and $\mathbf{A}=[\mathbf{1}\ \mathbf{x}\ \mathbf{y}]$ with **1** being a column of ones. The strain elements are then

$$e_{ux} = a_1\,, \quad e_{uy} = a_2\,, \tag{4.21}$$

$$e_{vx} = b_1\,, \quad e_{vy} = b_2\,. \tag{4.22}$$

Letting $\tilde{\mathbf{N}}$ denote the reduced normal equation matrix with a_0 or b_0 eliminated, the elements of the strain matrix can be expressed together in vector form as

$$\mathbf{e} = \begin{bmatrix} e_{ux} \\ e_{uy} \\ e_{vx} \\ e_{vy} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{N}}^{-1} \mathbf{A}^T \mathbf{u} \\ \tilde{\mathbf{N}}^{-1} \mathbf{A}^T \mathbf{v} \end{bmatrix} = \mathbf{Q}\,\boldsymbol{\delta}, \tag{4.23}$$

where the local displacement field vector is ordered as $\boldsymbol{\delta}^T=(\mathbf{u}^T, \mathbf{v}^T)$.

4.4 Strength Analysis Using Strain

The use of strain to analyse the strength of a geodetic network was first proposed by Vaníček et al. [1981] and later developed by Dare [1983]. Rather than describing the ability of a network to resist the propagation and accumulation of random errors, the strain approach is based on the ability to resist the propagation and accumulation of systematic errors or blunders (i.e., changes of a non-random nature).

The basic approach is to perform a series of separate strain analyses by individually changing the observation values. Each such perturbation produces a new displacement field and thus strain at each point. The most realistic results were obtained when changing each observation by its standard deviation [Dare, 1983]. A measure of strength is obtained by assuming the network is only as strong as its weakest link. The weakest link corresponds to the largest strain parameter at each station from the entire series of strain solutions for all observation perturbations.

With this technique, a virtual displacement field must be generated for every observation in the network. Although this may seem like a daunting task, sequential estimation methods can be used to advantage here (see Craymer et al. [1989]). The displacement field $\boldsymbol{\delta}$ in response to a change of an observation can then be given directly in terms of the perturbed observation vector $\Delta\mathbf{l}$, which contains only one non-zero element equal to the standard deviation of the observation; i.e.,

$$\boldsymbol{\delta} = -\mathbf{N}^{-1} \mathbf{A}^T \mathbf{P}\, \Delta\mathbf{l} = \mathbf{T}\, \Delta\mathbf{l}\,, \tag{4.24}$$

where $\mathbf{A}$ is the design matrix, $\mathbf{P}$ is the weight matrix of the observations and $\mathbf{N} = \mathbf{A}^T \mathbf{P} \mathbf{A}$ is the normal equation matrix. Because only one observation is changed at a time, only one column of the normal equation matrix is needed to evaluate the displacement field if the observations are independently weighted. Since the strain elements are linear functions of the displacements (see eqn. (4.23)), we can write the following system of linear equations for each change to an observation:

$$\mathbf{e} = \mathbf{Q}\,\boldsymbol{\delta} = \mathbf{Q}\,\mathbf{T}\,\Delta\mathbf{l} = \mathbf{R}\,\Delta\mathbf{l}\,. \tag{4.25}$$

We are now interested only in the largest deformations at each point as measured by the deformation primitives: dilation σ, total shear γ, and differential rotation ω (cf. eqns. (4.5), (4.13) to (4.15) and (4.4)). New deformation primitives are computed one at a time for a change in each observation by its standard deviation. Only the largest primitives (in absolute value) at each point are retained as a measure of the weakest link. These maximum values (denoted by σ_{max}, γ_{max}, and ω_{max}) at each point in the network describe the network strength and are referred to as strength in scale, strength in shear, and strength in rotation (twist), respectively. They can be displayed as contour plots or 3D surface plots. Only the former is currently supported in the NETAN program [Craymer et al., 1988; 1989].

4.5 Datum Independence of Strength

The effect of the coordinate system or datum definition on the computed strain parameters is an important issue in the strength analysis of networks. Datum is taken here to mean the definition of the origin and orientation of the coordinate system as well as the scale. The origin is usually defined by specifying fixed or heavily weighted coordinates for one or several points in the network. The orientation is often defined using weighted observations such as azimuths or observed position differences between points. The scale is generally defined using weighted distances or, again, position difference observations. Two or more weighted position observations can also be used to define datum orientation and scale.

Ideally the strength of a network should not depend on the choice of a datum so that different people analysing the same network, but using different datums, will get the same strength parameters. It is shown here that rotations and scale changes have only a very small and negligible effect on the strength parameters and that translations of the datum origin have no effect at all. The effects of translations, rotations, and scale changes on strength parameters will be evaluated in terms of strain parameters only, strength being just the largest strain parameter at each station resulting from a series of virtual displacement fields.

It is important to bear in mind that only one datum definition is used in a single strength analysis. The virtual displacement fields generated for the strength analysis are due only to changes in the network observations ('blunders') and not to changes in the datum. The question is whether a displacement field generated by such a 'blunder' gives the same strain as another displacement field also generated by the same 'blunder' but using a different datum. In practice, the differences in datums are likely to be very small; say, less than a degree in the orientation of the coordinate axes, and a few hundred parts per million in scale.

Translations of Datum Origin

Differences in the datum origin between different strain (or strength) solutions completely cancel in the determination of the displacement field. That is, the displacement fields for both solutions are identical even though they may be based on datums with different origins.

This can be proven very easily by considering one strain solution where $\mathbf{x}_1$ are the original coordinates of points in the local displacement field and $\mathbf{x}_2$ are the coordinates after the network has been perturbed by a single blunder. The local displacement field $\boldsymbol{\delta}$ is then

$$\boldsymbol{\delta} = \mathbf{x}_2 - \mathbf{x}_1 \, . \tag{4.26}$$

Consider now a second strain solution using a different datum origin which is offset from that for the first solution by a translation $\Delta\mathbf{x}$. For this solution, the original coordinates $\mathbf{x}_1^*$ and those $\mathbf{x}_2^*$ after perturbation by the same blunder can be expressed in terms of the coordinates for the first solution as

$$\mathbf{x}_1^* = \mathbf{x}_1 + \Delta\mathbf{x} \, , \tag{4.27}$$

$$\mathbf{x}_2^* = \mathbf{x}_2 + \Delta\mathbf{x} \, . \tag{4.28}$$

The displacement field $\boldsymbol{\delta}^*$ for this second solution is then

$$\boldsymbol{\delta}^* = x_2^* - x_1^* = x_2 - x_1 = \boldsymbol{\delta} \,, \tag{4.29}$$

which is identical to that for the first solution. Any translation of the datum origin therefore cancels in the virtual displacement field. Since the virtual displacement fields are identical in both strain solutions, the strain and strength parameters also will be identical. Strain and strength parameters are therefore invariant to translations of the datum origin.

It is important to point out here that strain is also invariant to displacements resulting from the translation of all points in a network. The 2D displacement fields in this case will have constant values for both the u and v components at all stations (i.e., they will be horizontal planes). The gradients (strain) of these displacement fields will be zero since a horizontal plane will have zero slope. This is the reason that strain is preferred in studies of crustal motion where it is not known whether the point fixed in a previous adjustment has moved.

Rotations of Datum Coordinate Axes

A change in the orientation of the coordinate axes defining the datum in a strain (or strength) analysis results in only a very small and negligible effect on the resulting strain and strength parameters. Given the same displacement field $\boldsymbol{\delta}$ for the first strain solution as above (generated by a blunder), the strain matrix **E** is defined by

$$\mathbf{E} = \mathrm{grad}(\boldsymbol{\delta}) \,. \tag{4.30}$$

Consider a second solution where the coordinate system has been rotated by an arbitrary rotation matrix **R**. The new coordinates before and after perturbation by the same blunder are, respectively,

$$x_1^* = \mathbf{R}\, x_1 \,, \tag{4.31}$$

$$x_2^* = \mathbf{R}\, x_2 \,. \tag{4.32}$$

The displacement field $\boldsymbol{\delta}^*$ for this second solution is then

$$\boldsymbol{\delta}^* = x_2^* - x_1^* = \mathbf{R}\, x_2 - \mathbf{R}\, \mathbf{x1} = \mathbf{R}\, \delta \,. \tag{4.33}$$

Note that for a small angle of rotation, the term $\mathbf{R}\,\boldsymbol{\delta}$ is only a second-order effect. The corresponding strain matrix $\mathbf{E}^*$ is

$$\mathbf{E}^* = \text{grad}(\boldsymbol{\delta}^*) = \mathbf{R}\,\text{grad}(\boldsymbol{\delta}) = \mathbf{R}\,\mathbf{E}\,. \tag{4.34}$$

If the rotation angle $\Delta\alpha$ is small, the rotation matrix for a single rotation about, e.g., the z-axis can be simplified to

$$\mathbf{R} \approx \begin{bmatrix} 1 & \Delta\alpha & 0 \\ -\Delta\alpha & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{I} + \begin{bmatrix} 0 & \Delta\alpha & 0 \\ -\Delta\alpha & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \mathbf{I} + \Delta\mathbf{R}\,. \tag{4.35}$$

The strain matrix in this new datum is then

$$\mathbf{E}^* = \mathbf{R}\,\mathbf{E} \approx \mathbf{E} + \Delta\mathbf{R}\,\mathbf{E} = \mathbf{E} + \Delta\mathbf{E}\,. \tag{4.36}$$

The effect of a change in datum orientation is therefore $\Delta\mathbf{E}=\Delta\mathbf{R}\,\mathbf{E}$. For strength analyses of geodetic networks, the changes to this will result in only a very small effect which will be negligible in all practical cases.

A worst case effect can be estimated by considering a solution with very large strains of about $e=1\text{x}10^{-4}$ (100 ppm). If the datum is changed by a rotation of the coordinate system by a large amount, say $\Delta a=1\text{x}10^{-2}$ radians (over half a degree), the change in the strain matrix from the first solution is only $\Delta a{\cdot}e=1\text{x}10^{-6}$ (1 ppm). In a strength analysis, the strain elements are unlikely to exceed 50 ppm in which case a rotation of over 1° will be required to produce a 1 ppm effect on strain. In practice, the orientation of the network datum will generally be known to much better than 1 degree accuracy. These estimates have been verified using numerical tests.

Changes of Datum Scale

The effect on strain (and strength) parameters due to changes in datum scale can be determined in a similar manner. In this case, the strain solutions before and after a change in scale of Δs results in the following coordinates

$$\mathbf{x}_1{}^* = (1 + \Delta s)\,\mathbf{x}_1\,, \tag{4.37}$$

$$\mathbf{x}_2{}^* = (1 + \Delta s)\,\mathbf{x}_2\,. \tag{4.38}$$

The displacement field $\boldsymbol{\delta}^*$ for this second solution is then

$$\boldsymbol{\delta}^* = \mathbf{x}_2{}^* - \mathbf{x}_1{}^* = (1 + \Delta s)\,\boldsymbol{\delta}\,. \tag{4.39}$$

and the corresponding strain matrix $\mathbf{E}^*$ is

$$\mathbf{E}^* = \mathrm{grad}(\boldsymbol{\delta}^*) = (1{+}\Delta s)\,\mathrm{grad}(\boldsymbol{\delta}) = (1{+}\Delta s)\,\mathbf{E} = \mathbf{E} - \Delta s\,\mathbf{E}\,. \tag{4.40}$$

Note that $\Delta s\,\boldsymbol{\delta}$, and thus $\Delta s\,\mathbf{E}$, is again only a second-order effect.

A worst case effect can also be estimated by again considering a solution with very large strains of about $e{=}1\mathrm{x}10^{-4}$ (100 ppm). If the datum is changed in scale by an extremely large amount, say $\Delta s{=}1\mathrm{x}10^{-2}$ (10 000 ppm), the change in the strain matrix from the first solution is only $\Delta s{\cdot}e{=}1\mathrm{x}10^{-6}$ (1 ppm). In a strength analysis, the strain elements are unlikely to exceed 50 ppm in which case a scale change of over 20 000 ppm will be required to produce a 1 ppm effect on strain. In practice, the orientation of the network datum will generally be known to much better than 10 ppm accuracy which would result in scale effects of only 0.001 ppm for this example. These estimates have been verified using numerical tests.

5. ROBUSTNESS ANALYSIS

5.1 Merging Reliability and Geometrical Strength Analysis

In the 20 plus years since Baarda [1968] proposed the concept of reliability analysis, the technique has found, albeit quite slowly, many proponents and followers. Based on a rigorous statistical foundation, the technique offers an alternative tool for analysing networks of various kinds, e.g., geodetic, photogrammetric , and those for engineering surveys. The only problem with reliability analysis is that the interpretation of its results, particularly those pertaining to positions as opposed to observations, is rather difficult. We can quantify the maximum expected observation residual that can escape purging as an outlier (blunder) by the standard statistical test for outliers. What we cannot learn from the analysis is just how much damage (distortion) such an undetected error (possible blunder) can cause to the network. On the other hand, there is a global indicator of 'external reliability' (equation (3.26)) provided in Baarda's technique, but this is far too coarse a measure to be of much real use. What is really needed is a much finer measure, commensurate in its fineness with the distinguishing power of the internal reliability, that would pinpoint areas, or even better, points, where one can expect the damage to be significant and other points where the damage should be expected to be insignificant. The individual indicators of 'external reliability' (equation (3.25)) associated with individual points provide a fine enough measure but they are datum dependent and there are far too many of them to be practical.

Some 10 years ago, work on one such measure started at UNB and culminated in 1983 with Dare's [1983] formulation of 'geometrical strength analysis' (GSA). This technique approaches the problem of network deformability or lack of it, i.e., strength, from a purely geometrical point of view. The question GSA answers is: How could a geodetic network deform in the worst case if the observations were burdened with some undetected non-random errors? The answer includes a somewhat unexpected complication — there does not exist one single scalar measure of such a deformation, but three independent measures. In Chapter 4, these three measures are called the 'deformation primitives.' They are the pure strain (scale), the total shear (shape), and the differential rotation (twist). Every one of these primitives shows one independent aspect of network deformation.

In Dare's formulation of GSA and the later application program NETAN [Craymer et al., 1988], little attention was paid to possible values of observation errors that could cause the virtual deformation analysed by GSA. Values equal to the standard deviations of individual observations were used to generate the studied deformation. GSA thus starts where Baarda's reliability analysis ends. This became obvious to us at the outset of this contract, and we decided to put these two techniques together to obtain a full image of the strength of a geodetic network. All that is required to 'marry' the two techniques is to take the maximum errors (blunder) undetectable by the standard statistical tests for outliers as estimated by the reliability analysis and use them as values that can cause the virtual deformation of the network in GSA. The result is that equation (4.25), which generates the vector of displacement gradients (strains), changes to

$$\mathbf{e} = \sqrt{\lambda_0}\mathbf{R}\boldsymbol{\sigma}^* \ , \tag{5.1}$$

where

$$\sigma_i^* = \frac{\sigma_i}{\sqrt{r_i}} \ . \tag{5.2}$$

In other words, this equation is created by substituting $\nabla\ell_i$ from equation (3.20b) for σ_i in equation (4.25). The subsequent treatment of **e** remains the same as in GSA.

5.2 Properties of Robustness Analysis

The resulting analysis, which combines the statistical aspects of Baarda's theory with the geometrical aspects of GSA, gives an answer to the proper question that one must ask if analysing the strength of a network; namely, What would be the worst possible deformation of a network whose observations had been tested for outliers (and detected outliers purged) on a specific confidence level $1-\alpha_0$? The analysis — we call it 'network robustness analysis,' to reflect contemporary statistical terminology where robustness means insensitivity to blunders — gives a picture of the network's potential worst deformation in terms of the three independent deformation primitives. Clearly, a high value of one of the primitives associated with a point, or a region, shows a weakness (in resistance to deformation) of the network at that point, or region, in the sense of that particular primitive (aspect). Low values, on the other hand, are indicative of good resistance to deformation, i.e., indicative of strength.

For the purpose of designing a network that would meet specific strength criteria, it would be necessary to formulate meaningful tolerance limits for admissible weaknesses in the three independent primitives: scale, shear, and twist. In other words, for specifications dealing with a design of desirably strong networks, it will be necessary to come up with a specific value of λ_0, which scales the three indicators of strength. (As we see from equation (5.1), $\sqrt{\lambda_0}$ is a common scale factor to all the results obtained from the robustness analysis.) In Chapter 3, it was shown that λ_0 is a function of α_0, the significance level selected for testing adjusted observations for outliers, and β_0, the probability of Type II error in the testing. While α_0 is selected beforehand, prior to the outlier testing, β_0 is free but should be specified for the purpose of choosing tolerance limits in the robustness specifications.

Two types of singularities can appear in a network subjected to robustness analysis: a geometrical singularity, and a singularity due to no redundancy. The first singularity is caused by specific geometrical configurations when the point to be analysed is either connected only to one other point, or when it is colinear with all the connected points. This type of singularity does not have anything to do with the strength of the network at that point; strength simply cannot be (reliably or at all) determined at that point. In the enhanced NETAN (see Chapter 6), strength indicators at these singular points are simply not plotted at all.

The other type of singularity occurs at points attached to the network by observation(s) whose redundancy number equals zero, i.e., at points whose position is derived from the minimal number of observations with no redundancy (e.g., two intersecting directions).

Because such observations are not checked at all by the test for outliers, there is no guarantee that such observations are not burdened with huge blunders and the point in question represents a point of infinite weakness — zero strength — in the network. The strength indicators at these points show very large values.

Finally, we wish to note here that robustness analysis is datum independent. The proof for the independence was shown in the previous chapter for the geometrical strength analysis and it fully applies here as well. The consequence of this property is that any choice of a minimally constrained adjustment model will yield the same results as far as network strength is concerned. It must be emphasized, however, that a network adjusted with some weighted constraints, e.g., a network for which positions of some points, together with their errors, are known a priori, cannot be viewed in the same light. Weighted constraints become an indivisible part of the network itself as much as the observations are, and the strength of such a network is as much affected by the constraints (and their weights) as it is by the observations (and their weights). The fact that the weighted constrains may also supply a datum for the adjustment is only incidental.

5.3 Comparison of Robustness and Geometrical Strength Analyses

To demonstrate the difference between the GSA and the robustness analysis, results of both analyses applied to the HOACS 3D synthetic network, are shown here. For a full discussion of the HOACS 3D network and its robustness, the reader is referred to Chapter 7.

Figures 5.1 and 5.2 show strength in scale as estimated by both techniques. The results are somewhat similar insofar as the extreme values are concerned: the maxima (in absolute value) are located at the same points, the northeast and the southwest corners, i.e., the weaknesses of the network have been pinpointed by both techniques the same way. On the other hand, the details are quite different and so are the magnitudes, as one should expect.

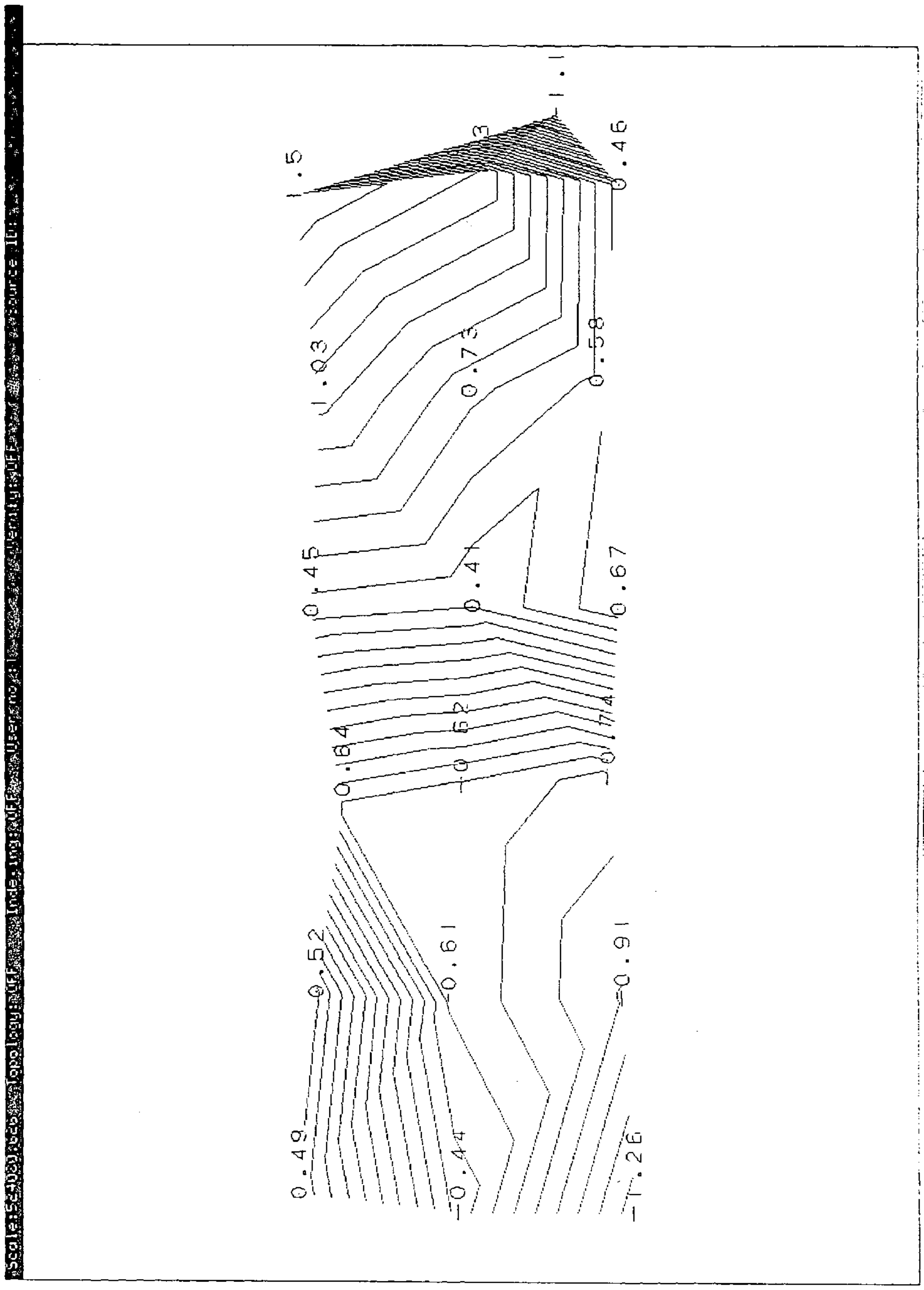

Figure 5.1 Geometrical strength in scale (ppm).

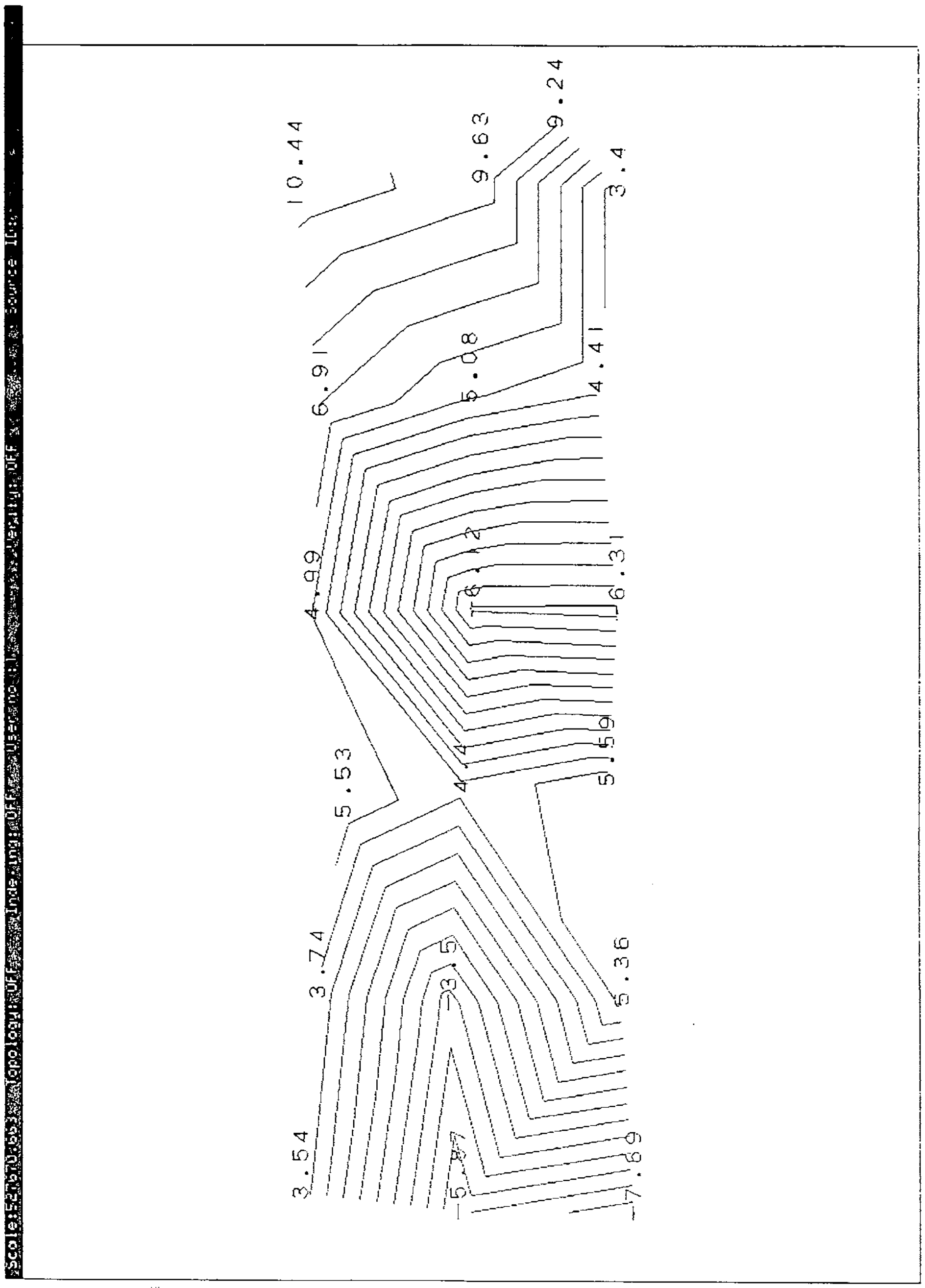

Figure 5.2 Robustness in scale (ppm).

Figures 5.3 and 5.4 show strength in shear. Once again, the point of extreme weakness is placed at the easternmost part of the network equally by both techniques. In the other aspects, the two plots are different.

Figures 5.5 and 5.6 display strength in twist. This time even the locations of extreme weaknesses are different; so much so that the point at the westernmost end of the network is at once identified as being the geometrically strongest, yet showing the least robustness. Interestingly, all the values of robustness in twist are negative, while geometrical strength shows both positive and negative values.

5.4 Comparison of Robustness and Covariance Analyses

In Chapter 2, we described what is generally understood by covariance analysis: the conglomerate of statistical tests based on the covariance matrices of the observations and of the adjusted positions (coordinates). This traditional analysis is based solely on statistical considerations and is concerned only with Type I error, random errors and their effect on the network. The effect is normally quantified by absolute and relative confidence regions and various derived quantities such as relative accuracies, whose purpose is to show just how much the network may be distorted (deformed) by the presence of errors presumed random.

In many countries, including Canada, this kind of analysis is the only one ever applied to geodetic networks. It forms the basis for Canadian federal specifications for horizontal control networks. The information contained in the covariance matrix of estimated positions is the only information about the 'strength of the network.' The effect of possible blunders that may escape detection by appropriate statistical tests is not considered and neither is the effect of geometrical weaknesses, at least not directly. The test that comes closest to a consideration of a blunder (but not its effect) is the one for outlier detection, where the result is assumed to be a set of blunderless observations. In brief, the covariance analysis deals with the second moments of the PDFs of observations and estimated parameters.

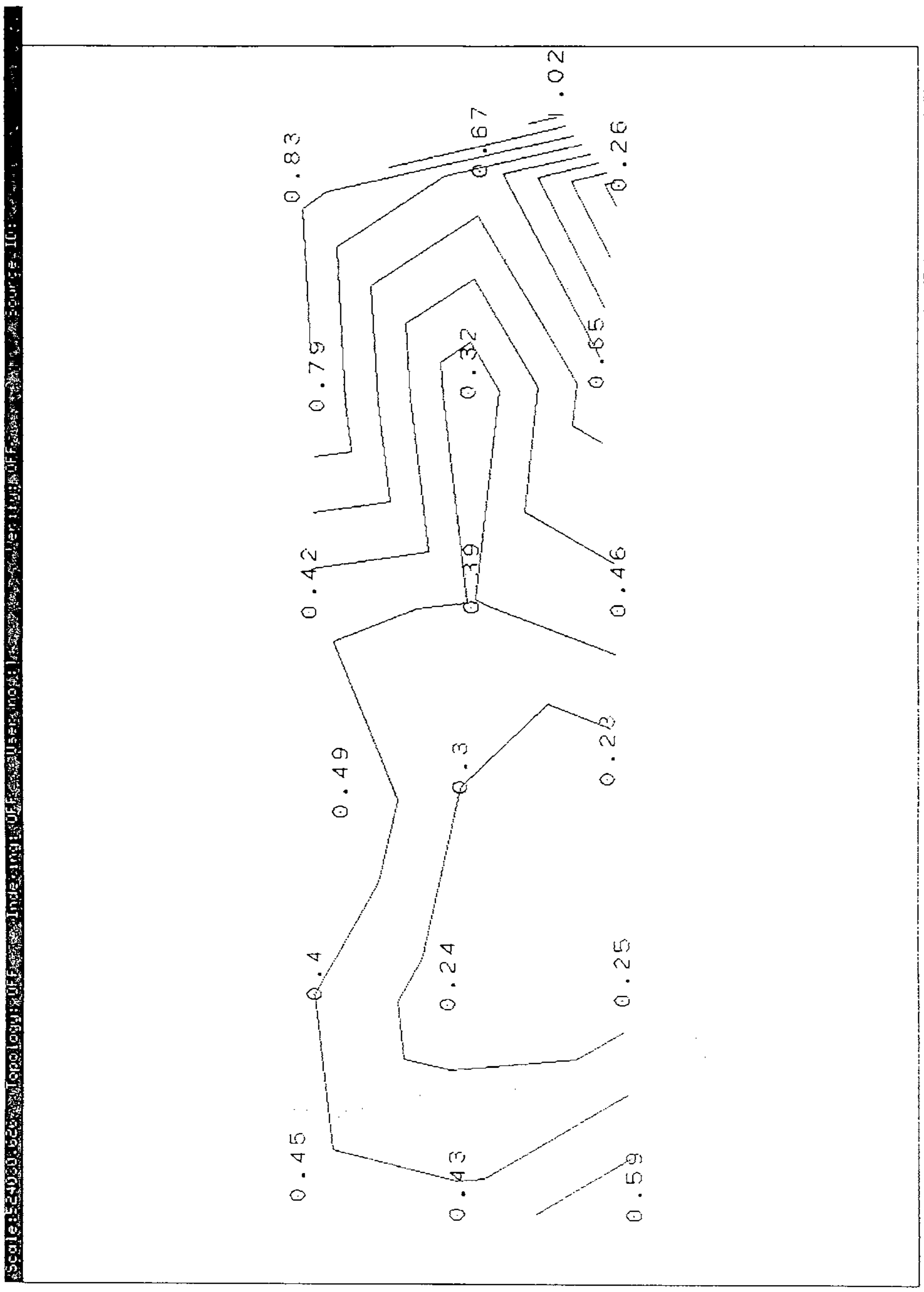

Figure 5.3 Geometrical strength in shear (ppm).

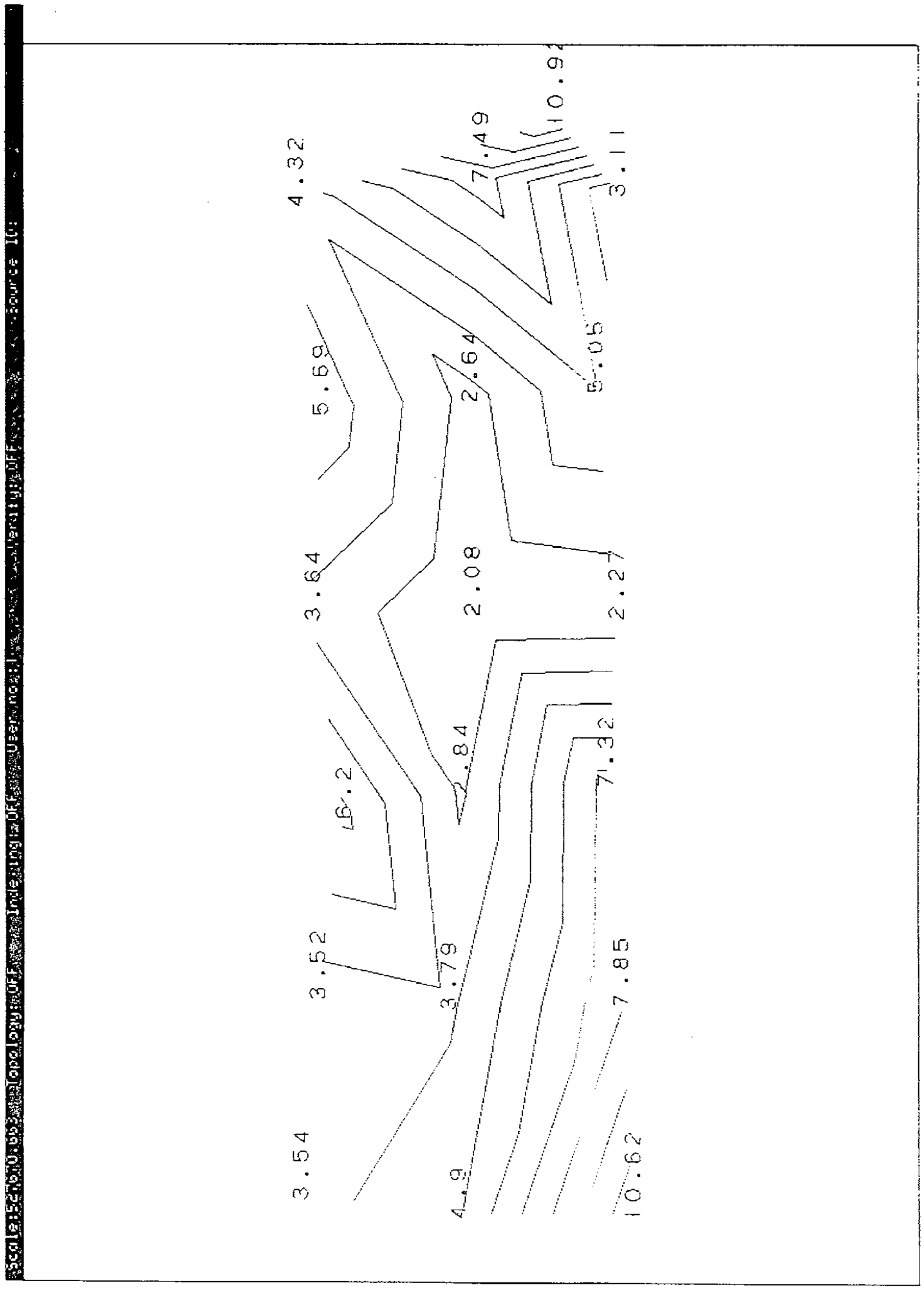

Figure 5.4 Robustness in shear (ppm).

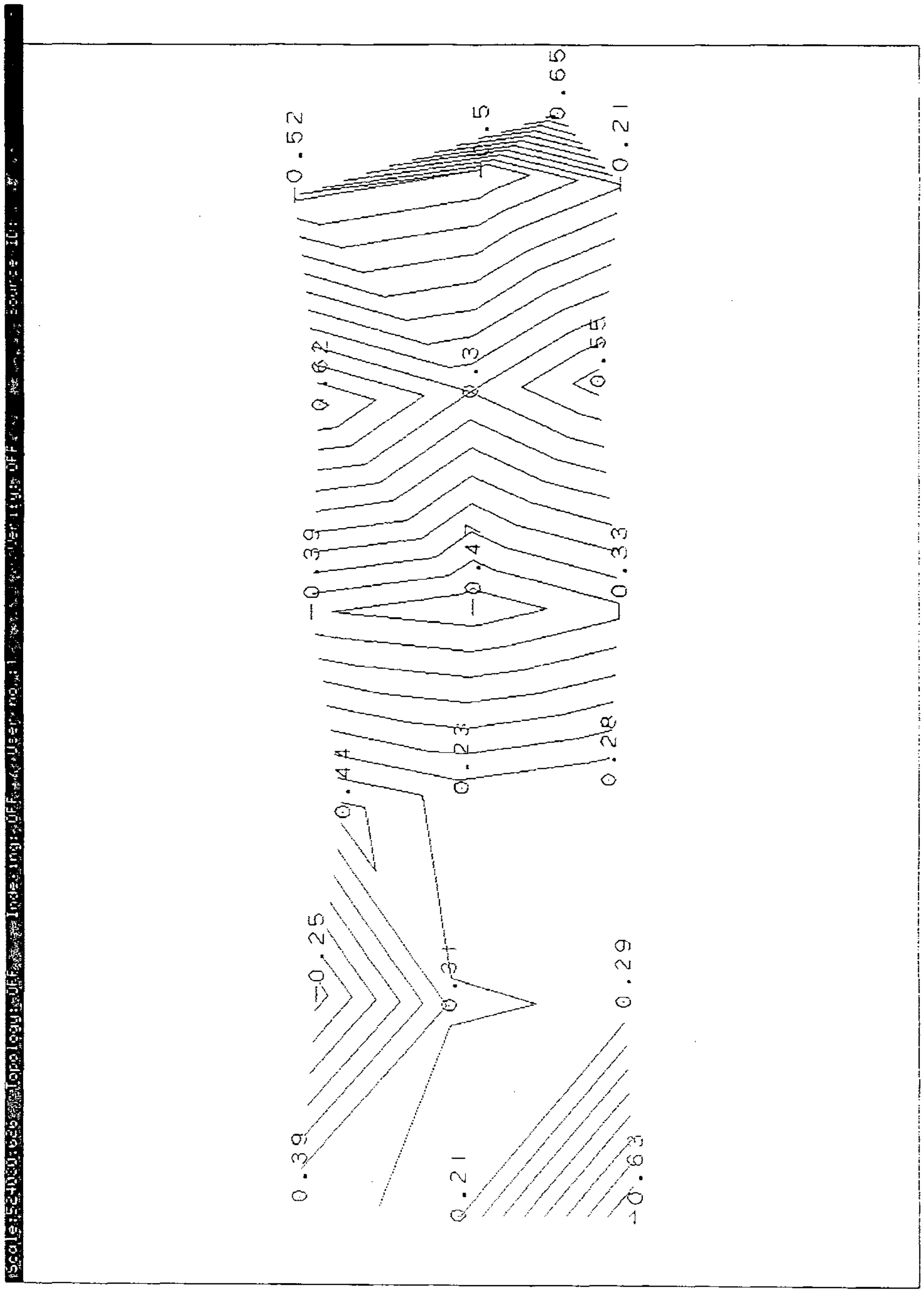

Figure 5.5 Geometrical strength in twist (ppm).

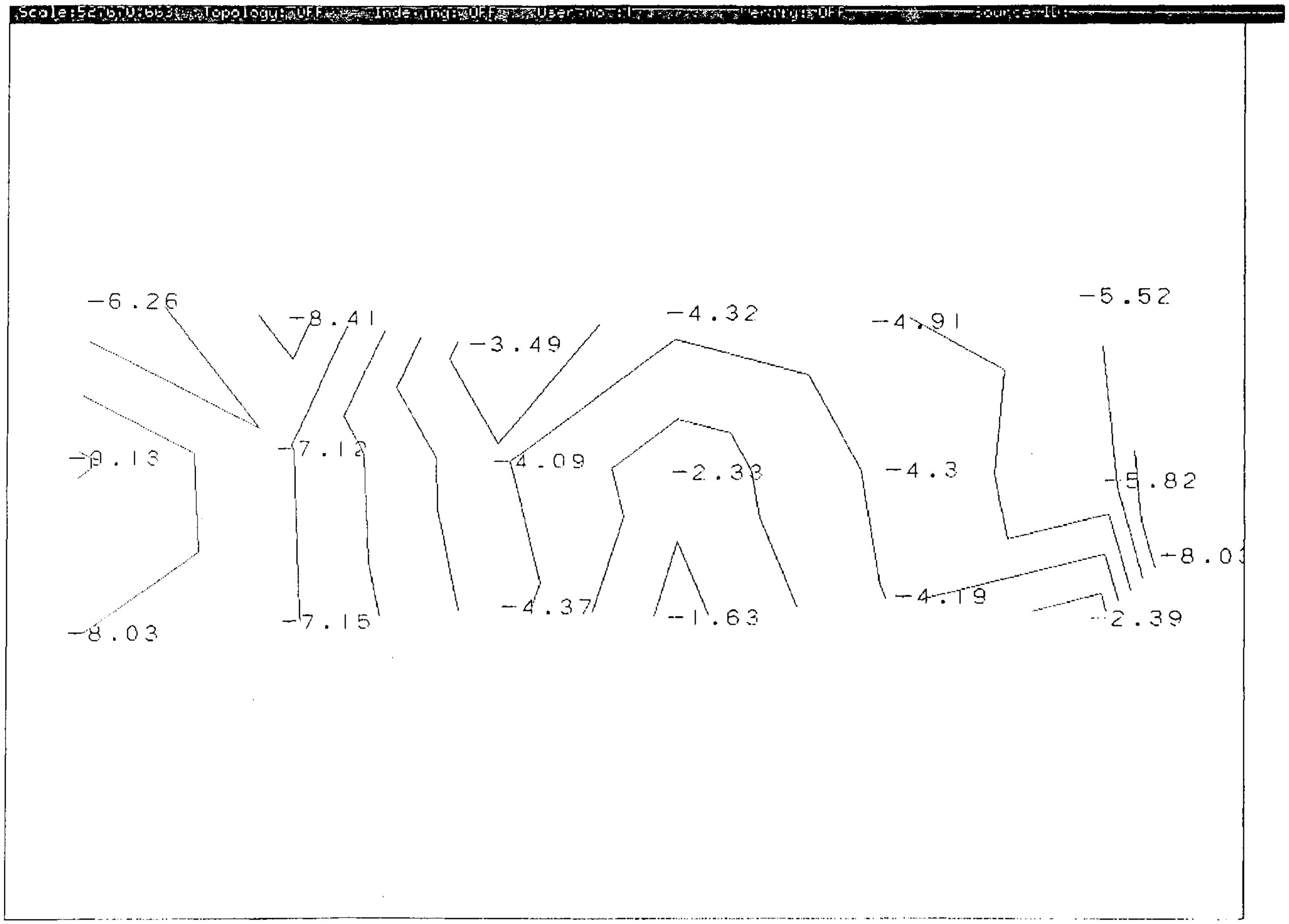

Figure 5.6 Robustness in twist (ppm).

Robustness analysis looks at the network from a different point of view, assuming that some blunders inevitably escape detection and make their presence felt by deforming the network. Hence it uses statistical tools also but only to estimate the magnitudes of potential undetectable blunders, i.e., it deals with first-order moments of the PDFs of observations. The rest of the analysis is purely deterministic (geometrical) and has no statistical connotation whatever. The picture of the network presented by the robustness analysis is what one should expect the strength to be: the ability to resist deformation, and should be understood as such; the robustness analysis is a strength analysis.

Clearly, the two analyses present quite different pictures of the analysed network. In some parts of the network, the messages from both analyses may be similar; in other parts, they may be very different, even contradictory. As an example, let us take the strength singularity caused by the lack of redundancy. In this case, the point in question will have zero strength while both absolute and relative confidence regions may not show any irregularity at all. Quite the contrary; if the single observation has a very low standard deviation, the pertinent relative confidence region will be very small, indicating a good network design.

To show the difference between the information contained in the results of either analysis, Figure 5.7 is included here. This figure is a plot of some relative confidence ellipses for the HOACS 3D network (for a full discussion of this network, see Chapter 7). This plot should be compared with Figures 5.2, 5.4, and 5.6 to see that the information content is quite different. One would be hard pressed to find any similarities between Figure 5.7 and any of the other three figures.

In some countries, e.g., Switzerland and Germany, Baarda's reliability analysis is used side-by-side with the standard covariance analysis. Robustness analysis should replace (complete) the reliability approach as a tool for analysing networks, side-by-side with the standard covariance analysis. As such, robustness analysis should make its way into network specifications to be used as a tool for network design, classification, and analysis.

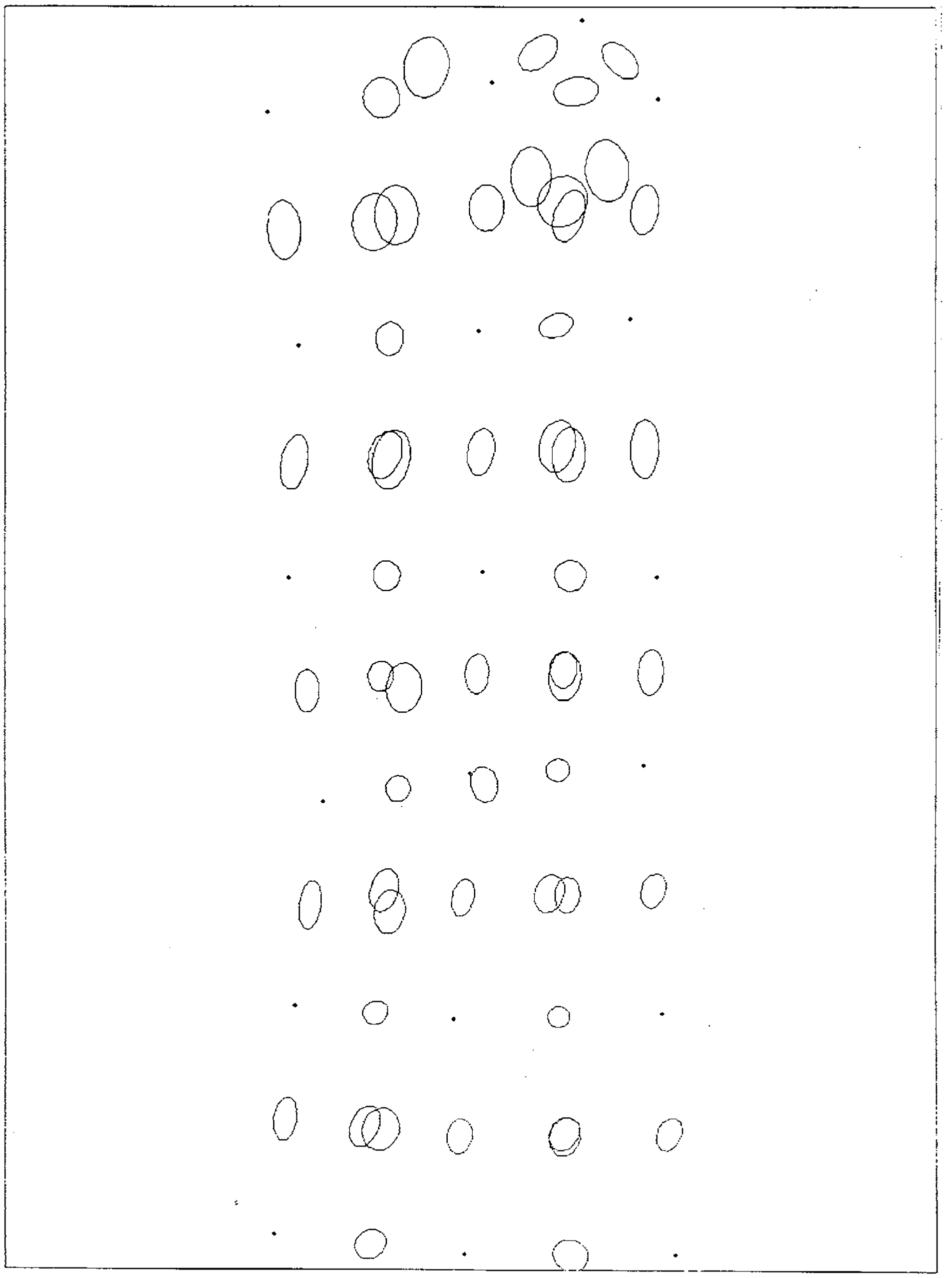

Figure 5.7 Relative confidence ellipses for the HOACS 3D network.

6. NETAN ENHANCEMENTS

6.1 Introduction

In this chapter, we discuss the incorporation of robustness analysis into the original NETAN software developed by Craymer et al. [1988; 1989]. Only a general description of the changes will be given. More details can be found in the source code.

The first modification was to replace the use of total strain with dilation to represent strength in scale. Total strain is computed from the square root of the sum of squares of the principal strains (eigenvalues) of the symmetric strain tensor. This quantity is a geometric mean and reflects only the absolute values of the principal strains. Negative changes in scale (i.e., contractions) can never be represented by this quantity. Dilation, on the other hand, is simply the sum of the principal strains (or the diagonal elements of the symmetric strain tensor) which accounts for the sign of the principal strains. The strength in scale based on this quantity can therefore represent both extension in scale (positive dilation) as well as contraction in scale (negative dilation).

The selection of the largest strain primitives to represent the strength parameters at each point in the network has also been changed. In the former version, the sign of the strain primitive was taken into account when searching for the largest value at each station. This resulted in large negative values being ignored in favour of small positive values. In the current version, this problem has been corrected by selecting the largest absolute value of the strain primitives representing the strength at each point.

The most significant change to NETAN involved the incorporation of robustness analysis. This necessitated computing the standard deviations of all residuals within NETAN since the GHOST software uses only Pope's approximation. Because these approximate values are identical for observations with the same standard deviation, they were of no use for determining the internal reliability; the redundancy numbers would (incorrectly) be the same for all observations with the same a priori standard deviation. Although the internal reliability results (i.e., the maximum undetectable error for each observation) are not presently included in the strength output, primarily because of the volume of information, this can be added in a later version if deemed necessary.

The practical implementation of robustness analysis required us to identify observations which are used to uniquely determine points in the network. For example, two intersecting directions to a point, two intersecting distances, or a distance and direction all provide only a unique determination of the relative position of a point. There is zero redundancy in these cases, and the maximum undetectable error is in theory infinite. Such observations are currently identified in the output listing and omitted from the strength computations for the purposes of this report. It is recommended, however, that this be changed in the next version of NETAN by assigning a specific value representing a very large number (say, 0.33E33) to the strength parameters at these stations. These points would then show up as very large peaks in the contour plots. Currently, these points are assigned zero strains and omitted from the plots.

A similar problem can also arise when a single observation is used to define the network datum. Examples are a single azimuth observation defining the network orientation or a single distance defining the network scale. A single position difference observation without any azimuths or distances defines both azimuth and scale. Large maximum undetectable errors result in these cases, which would overwhelm the strength results. Therefore these observations have also been omitted from the strength analysis for the purposes of this report. Currently, a tolerance limit of 0.001 for the redundancy number is used to detect such cases. Observations with redundancy numbers smaller than this will be identified in the output listing and omitted from the strength analysis. We recommend, however, that the software be changed to use the actual (large) maximum undetectable errors in the strength computations so that these points will show up as very large peaks in the contour plots of the strength parameters.

A more fundamental problem with the strength (and strain) computation occurs when there is only one unique observation tie to a station. For example, a direction and distance both along the same line. Such cases represent geometric singularities in the determination of strain since it will not be possible to fit a plane surface to just two points and no strain can therefore be computed. These strength and robustness parameters at points with such geometric singularities are currently assigned zero values. It is recommended instead to flag these points using a specific value representing an undefined quantity (say, 0.33E33) in the next version of NETAN.

Geometric ill-conditioning or near-singularities can also arise when the observation ties to a station are near collinear. The fitting of a plane to nearly collinear lines will be very ill-conditioned and may result in spuriously large strain elements. An example of this is shown for the real network analysed in Chapter 7. Presently NETAN does not identify such geometric ill-conditioning. One possibility may be to simply check the determinant of the normal equation matrix for the surface fitting solution. Small values would indicate geometric ill-conditioning. An alternative and more general approach may be to estimate standard deviations for the strain elements and strength parameters and to perform some elementary statistical tests for significance. It is strongly recommended to investigate these and other methods of identifying and dealing with such geometric ill-conditioning.

External reliability is not computed since the robustness analysis of the internal reliability provides a better and more detailed picture of the strength (deformation) of the network. Although this could be incorporated if desired, it would be very time consuming (of the same order as the robustness analysis). Nevertheless, external reliability may provide a means of providing strength estimates at points where there are geometric singularities or ill-conditioning.

6.2 Update to User's Guide

The incorporation of robustness analysis into the NETAN software resulted in a new "Strength Analysis Option" menu for selecting the type of strength analysis to be performed; either geometrical strength or robustness analysis (see Figure 6.1). The geometrical strength analysis uses the observation standard deviations to perturb the network as in the original version of NETAN. The robustness analysis perturbs the network using the internal reliability measure, i.e., the maximum undetectable error for each observation. This menu is presented immediately after selecting the "Strength Analysis" option from the main menu.

```
Strength Analysis Options
-------------------------
0)  Robustness analysis
1)  Geometrical strength analysis

Select option [0]:
```

Figure 6.1 "Strength Analysis Options" menu.

In addition to the prompts already described in Craymer et al. [1988], selection of the "robustness analysis" option also presents an additional prompt to enter the non-centrality parameter for the internal reliability computations. The following prompt is used to list the acceptable non-centrality values together with their associated levels of significance and power of the test:

```
Non-centrality parameters:
-------------------------
Significance    Power of the Test (%)
Level (%)        99.0  95.0  90.0  85.0  80.0
-----------     ------------------------------
   0.1           5.60  4.94  4.58  4.34  4.14
   1.0           4.90  4.22  3.86  3.62  3.42
   5.0           4.29  3.61  3.24  3.00  2.80

Enter non-centrality parameter:
```

Finally, the output listing for the strength analysis has also been augmented with additional information pertinent to the robustness analysis. Added to the original strength output listing is a list of the omitted observations which uniquely determine points or uniquely define the network datum.

7. NUMERICAL EXAMPLES

7.1 Introduction

In this chapter, examples of robustness analysis are given for both simulated and real three-dimensional (3D) networks, each with a variety of different types of geodetic observations including azimuths, directions, distances, 3D position observations, and 3D position difference observations.

The plots of the robustness results presented here have been generated in two ways. The plots of observation ties were created using NETAN. Hard copies were made from screen dumps to a Tektronix 4693D Color Image Printer using a thermal wax printing process. Plots of the robustness in rotation, shear, and scale were generated using the CARIS software system only for the purposes of this report. A temporary data file containing the information to be plotted was generated by NETAN for input to CARIS. CARIS allowed us to include on the contour plots the robustness values at each point in the network. NETAN does not display this information on the plot but instead lists station and observation information interactively when the user graphically selects a point or observation tie.

7.2 Simulated Network HOACS

A simulated network, called HOACS, was obtained from the Canadian Geodetic Survey. This test network was originally created by the U.S. National Geodetic Survey for testing their own network adjustment software and is also used by the Canadian Geodetic Survey to test their 3D adjustment software GHOST. The network consists of a variety of geodetic observations including:

- 11 free stations (none explicitly fixed)
- 3 azimuth observations with 1 second standard deviations
- 77 direction observations with 0.7 second standard deviations

- 6 distance observations with 2 ppm standard deviations
- 8 three-dimensional position observations with 10 cm standard deviations (i.e., 24 coordinate observations)
- 3 three-dimensional position difference observations with 5 mm standard deviations (i.e., 9 coordinate difference observations)
- 3 three-dimensional position difference observations with 1 cm standard deviations (i.e., 9 coordinate difference observations)

Figures 7.1 to 7.5 illustrate the locations of the stations and the different types of observations. Table 7.2 gives a listing of the input GHOST data file for this network.

The results from the NETAN robustness analysis are displayed in terms of robustness in rotation (local twisting), robustness in shear (local changes in configuration or shape), and robustness in scale in Figures 7.6, 7.7, and 7.8, respectively. The NETAN output listing for this analysis is given in Table 7.3. These robustness results are all based on α_0=5% and β_0=5% which gives a non-centrality parameter of $\sqrt{\lambda_0}$=3.61 (the standardized value of maximum undetectable error). Different α_0 and β_0 result in a different $\sqrt{\lambda_0}$ which only scales the magnitude of the strength parameters by the ratio of the non-centrality parameters. The plots are otherwise identical.

The results indicate that the perimeter of the network is relatively weaker than the middle since the largest values for rotation, shear, and scale are all located on the edges of the network. No points uniquely determined by a minimum number of observations are present. The datum orientation and scale are defined by a multitude of observations. Because of the ideal geometry of this network, all strain determinations were well conditioned. Table 7.1 summarizes the range of values, the largest and smallest (absolute) values in magnitude, and the average and standard deviation (dispersion about the mean) for each robustness parameter. Note that the averages and standard deviations of these parameters are all of the same magnitude as the relative accuracy of the observations. Individual values less than about 10 ppm may therefore not be very statistically significant (this should be investigated more rigourously by developing and implementing algorithms for determining the standard deviations of the strength parameters).

Table 7.1 Summary of robustness results for the HOACS network.

	Robustness in rotation (μrad)	Robustness in shear (ppm)	Robustness in scale (ppm)
Maximum	–1.6	10.9	10.4
Minimum	–9.1	2.1	–7.7
Largest absolute	–9.1	10.9	10.4
Smallest absolute	–1.6	2.1	3.4
Mean	–5.3	5.1	2.4
Standard deviation	2.2	2.6	5.9

The robustness in rotation results are illustrated in Figure 7.6 and listed in Table 7.3. These results describe the ability of the network to resist local changes in orientation (twisting). The largest values are obtained for station 1017 (–9.1 μrad due to position observation #117), station 1015 (–8.4 μrad due to position observation #117), station 1016 (–8.0 μrad due to position observation #117), and station NO SUCH MOUNTAIN (–8.0 μrad due to direction observation #8). All of these weak points are located on the perimeter of the network where there are fewer observations tying these stations to the rest of the network. Even the largest of these values, however, is well within first-order accuracy standards (20 ppm). The best (smallest absolute value) robustness in rotation is obtained for station 1007 (–1.6 μrad) which, although located at the bottom edge of the network, has many observations tying this point to the rest of the network. The average of the differential rotations is only –5.3 μrad and the standard deviation (dispersion about the mean) only 2.2 μrad.

The robustness in shear results are given in Figure 7.7 and Table 7.3. These results describe the ability of the network to resist local deformations in configuration or shape. The largest values are obtained for station NO SUCH MOUNTAIN (10.9 ppm due to direction observation #8) and station 1016 (10.6 ppm due to position observation #123), both of which are on the perimeter of the network and have fewer observation ties. These values are also well within first-order accuracy limits. The best (smallest absolute value) robustness in shear is obtained for station 1008 (2.1 ppm) which is in the middle of the network and has many observation ties connecting it to the rest of the network. The average shear is only 5.1 ppm and the standard deviation is 2.6 ppm.

The robustness in scale results are given in Figure 7.8 and Table 7.3. These results describe the ability of the network to resist local deformations in scale (dilation). The largest values are obtained for station 1003 (10.4 ppm due to position difference observation #90), station 1002 (9.6 ppm due to position difference observation #90), station NO SUCH MOUNTAIN (9.2 ppm due to position observation #114), and station 1016 (–7.7 ppm due to position difference observation #105). All of these stations are on the perimeter of the network and have fewer observation ties to the rest of the network. These values are again well within first-order accuracy limits. The best (smallest absolute value) robustness in scale is obtained for station 1001 (3.4 ppm) which, although at the edge of the network, has many observation ties (especially 3D position differences) to the rest of the network. The average dilation is only 2.4 ppm and the standard deviation is 5.9 ppm.

Other than the number of observations to and from the points of interest, there does not seem to be much indication of the cause of the weaknesses. Although different observations seem to affect different robustness parameters in some cases, there are other cases where these trends breaks down. We therefore have to rely on an analytical approach to learn about the weaknesses. More experience and experiments will be required to gain a better understanding of how the robustness parameters are affected by the different types of observations.

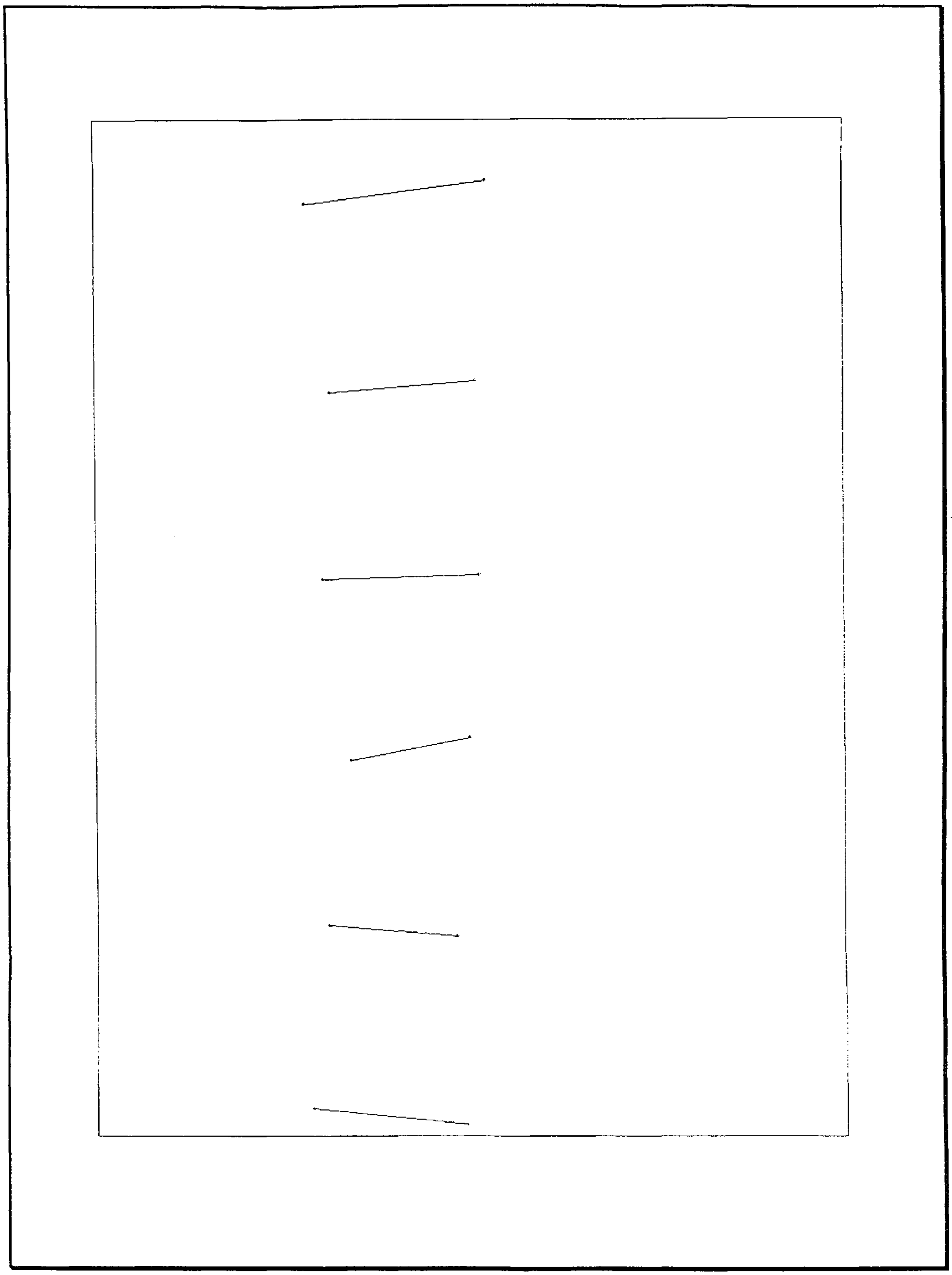

Figure 7.1 Distance observations for simulated 3D HOACS network.

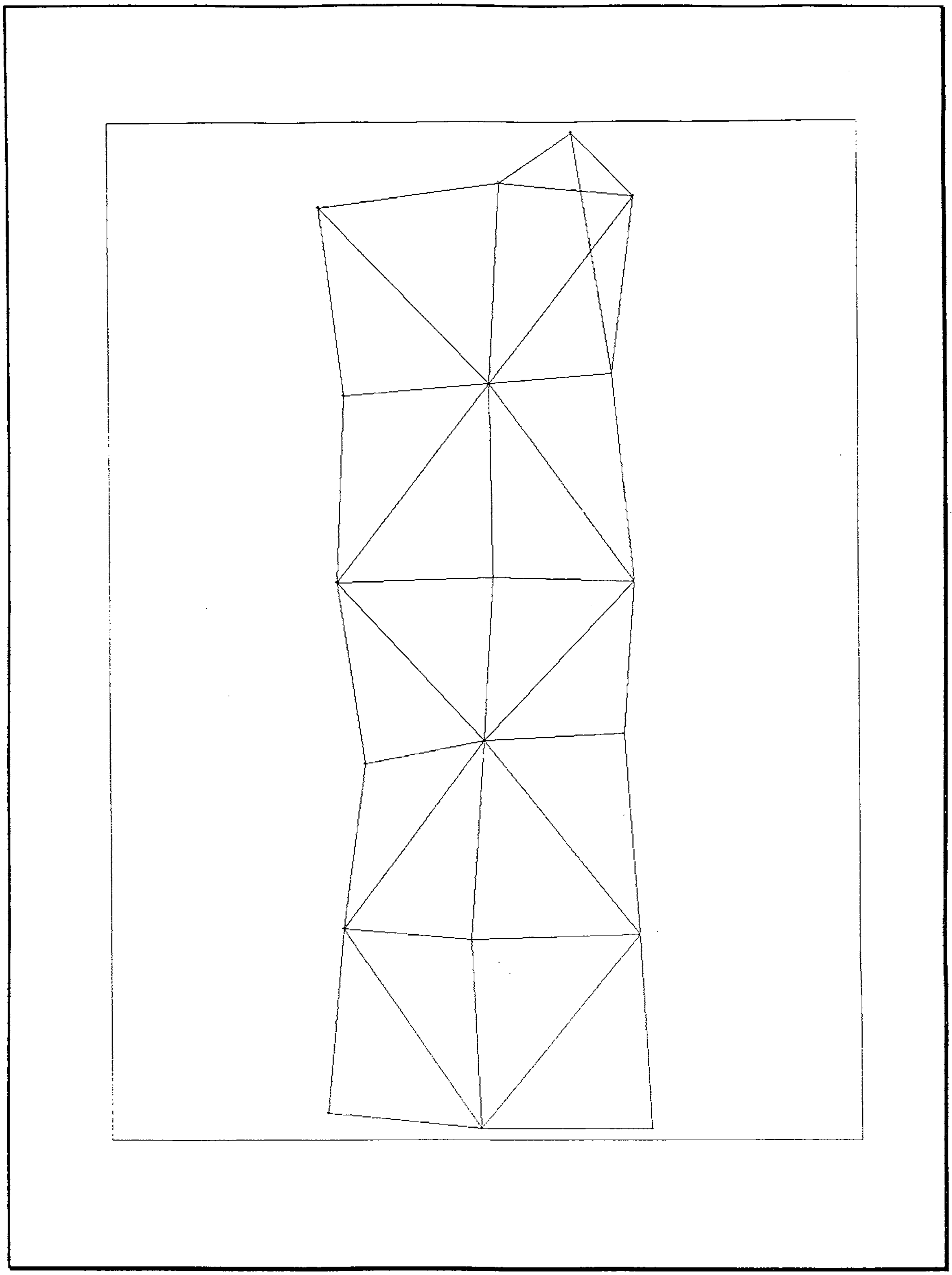

Figure 7.2 Direction observations for simulated 3D HOACS network.

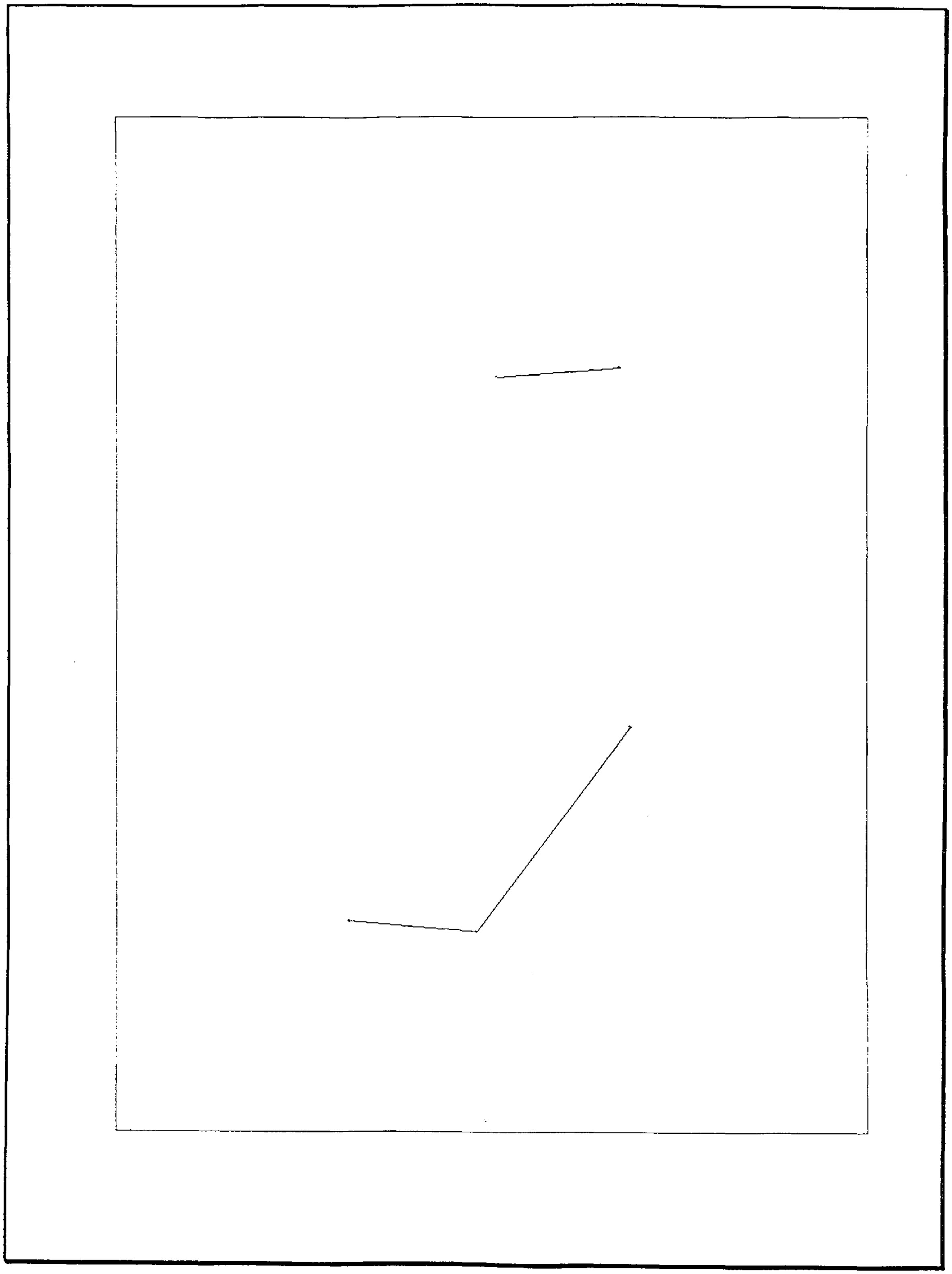

Figure 7.3 Azimuth observations for simulated 3D HOACS network.

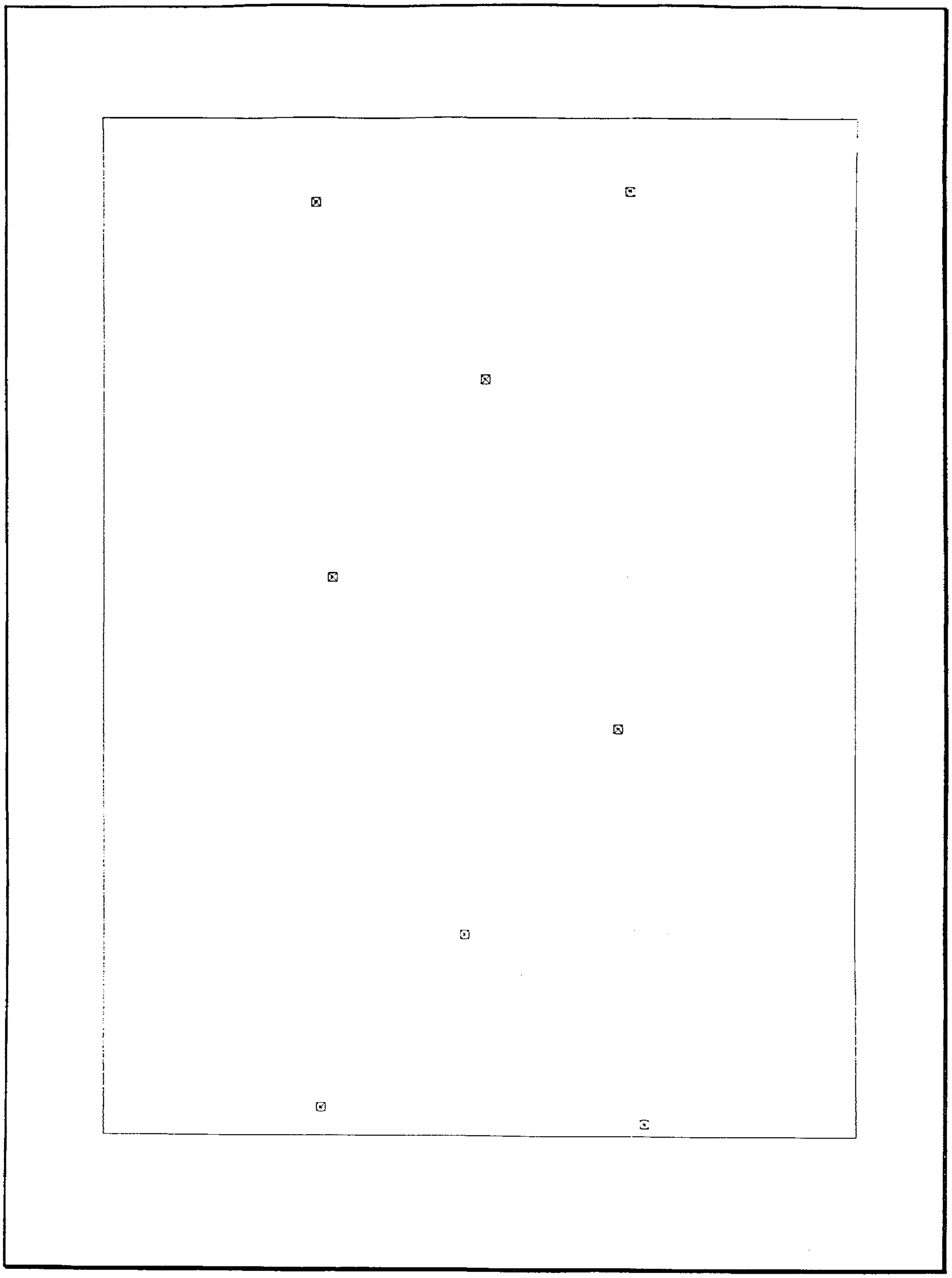

Figure 7.4 3D position observations for simulated 3D HOACS network.

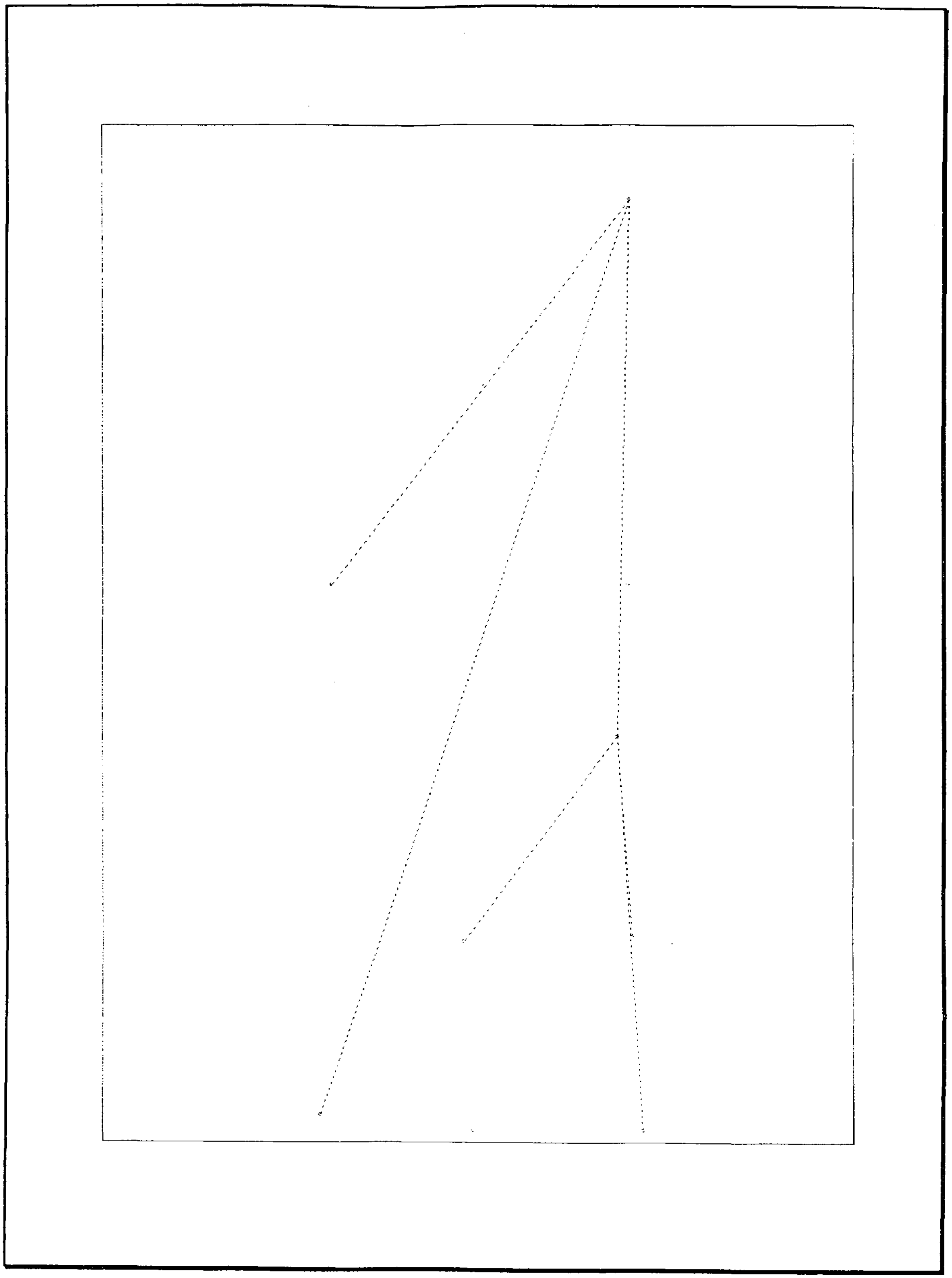

Figure 7.5 3D position difference observations for simulated 3D HOACS network.

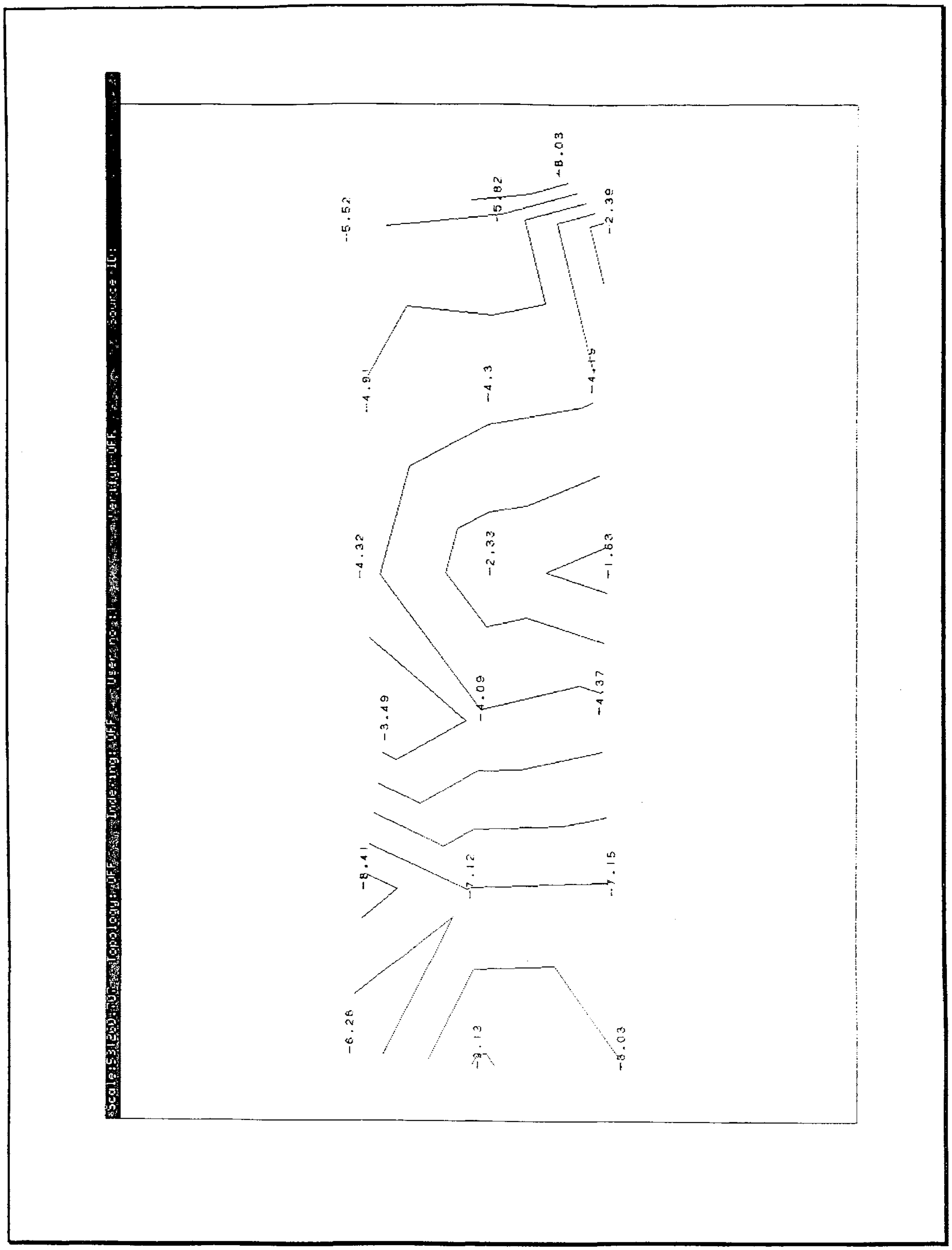

Figure 7.6 Robustness in rotation for simulated 3D HOACS network.

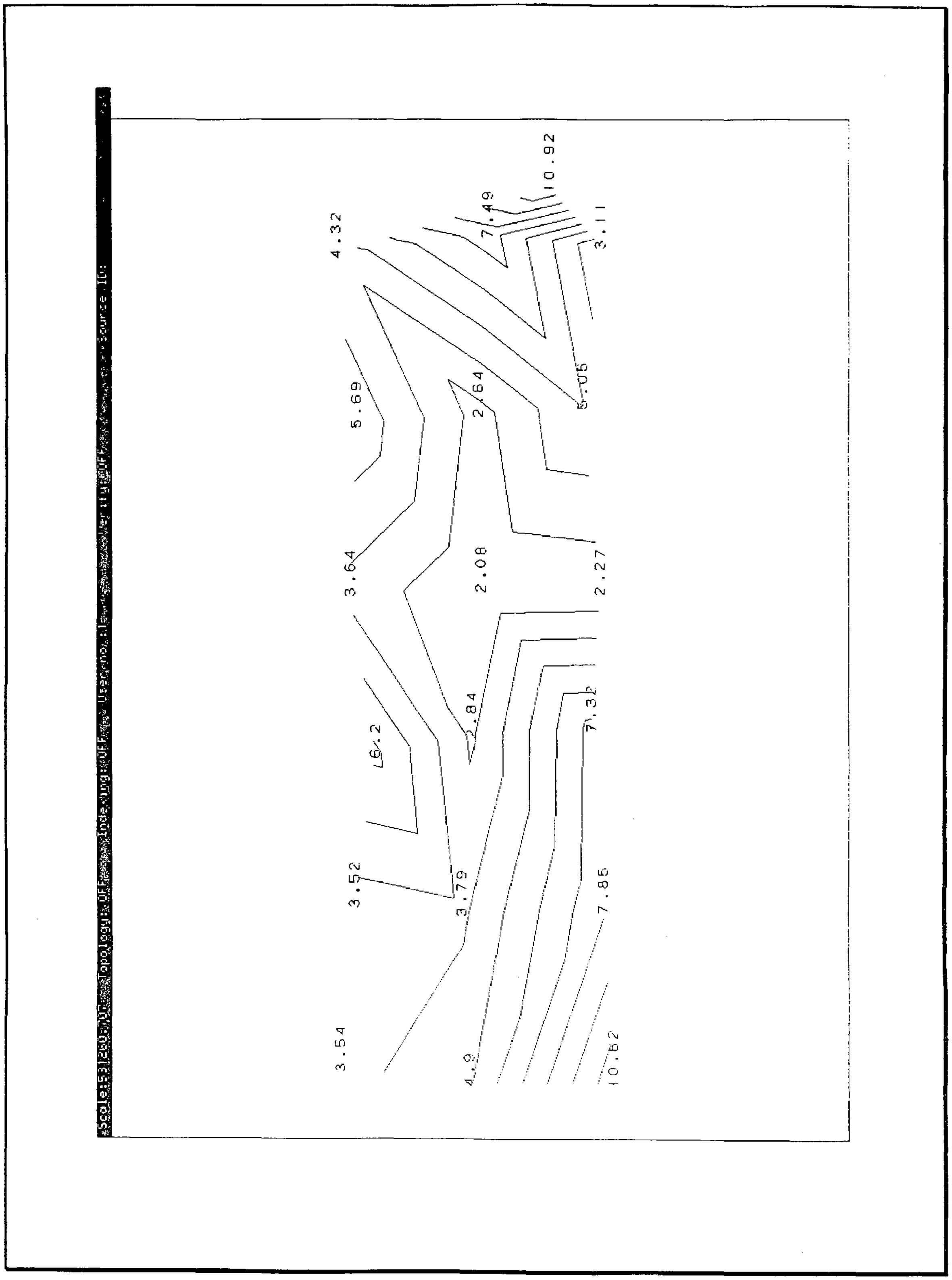

Figure 7.7 Robustness in shear for simulated 3D HOACS network.

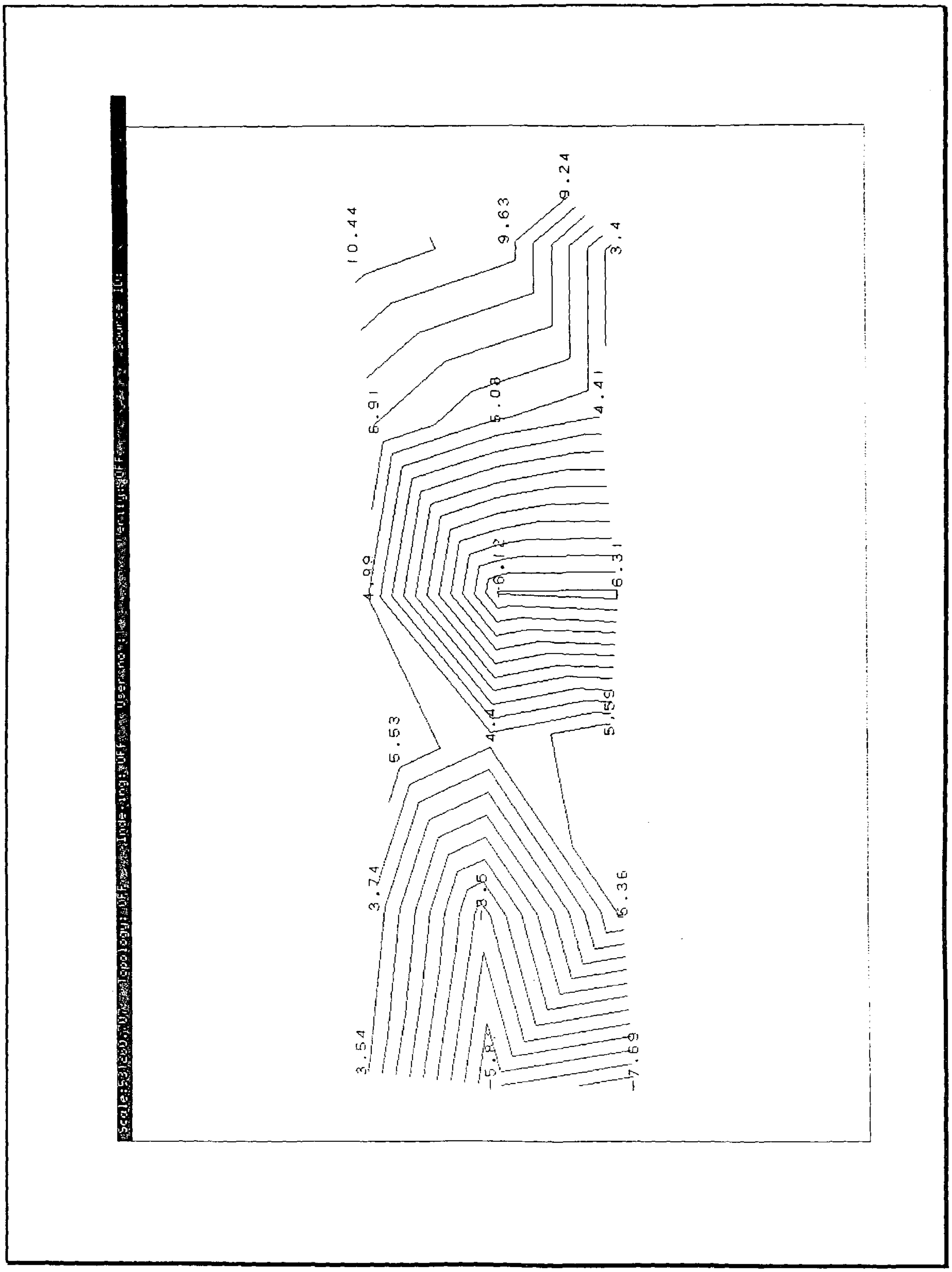

Figure 7.8 Robustness in scale for simulated 3D HOACS network.

Table 7.2 GHOST input data file for simulated 3D HOACS network.

```
HOACS TERRESTRIAL TEST DATA
  1 3   1   1 1 1 1
*
* STATION INFORMATION SECTION
*
 10
 4300NO SUCH MOUNTAIN           30  5  0.00000w 89 55  0.00000 3500.000
 43001001 STATION              30  0  0.00000w 90  0  0.00000 1955.800
 43001002 STATION              30 10 54.03925w 89 58 58.98067 2164.320
 43001003 STATION              30 25 40.03862w 90  0 54.04869 1996.800
 43001004 STATION              30  1 46.97851w 90 14  9.01720 2015.190
 43001005 STATION              30 11 47.98894w 90 14 58.98289 1914.260
 43001006 STATION              30 23 40.99617w 90 15 57.95561 2203.180
 43001007 STATION              30  0  1.94870w 90 30 52.04613 1937.020
 43001008 STATION              30 11 35.94621w 90 30 33.01558 2075.930
 43001009 STATION              30 24 16.94934w 90 30 56.02636 2042.170
 43001010 STATION              30  0 57.03795w 90 43   .04138 1987.610
 43001011 STATION              30 12 20.95802w 90 43 35.04939 1954.820
 43001012 STATION              30 22  1.99561w 90 45 25.95232 1874.160
 43001013 STATION              29 59 43.97387w 90 59 10.05335 1819.660
 43001014 STATION              30 13 29.02889w 90 59 34.00567 2006.180
 43001015 STATION              30 23 53.99599w 90 58 39.97635 1950.240
 43001016 STATION              29 58 51.95506w 91 15  3.97056 2150.690
 43001017 STATION              30 12 46.02388w 91 15  1.05684 2071.250
 43001018 STATION              30 25 15.02776w 91 13 42.95407 1990.500
*
* ASTRONOMIC DATA RECORDS
*
 7   1001 STATION            30     1.08   w 90     1.94000
 7   1002 STATION            30 10 56.48   w 89 59 01.10000
 7   1003 STATION            30 25 37.30   w 90 00 54.33000
 7   1004 STATION            30 01 45.69   w 90 14 15.54000
 7   1005 STATION            30 11 49.69   w 90 15 01.50000
 7   1006 STATION            30 23 38.59   w 90 16 02.12000
 7   1007 STATION            30     3.51   w 90 30 49.12000
 7   1008 STATION            30 11 36.81   w 90 30 36.41000
 7   1009 STATION            30 24 16.69   w 90 30 59.39000
 7   1010 STATION            30    55.32   w 90 43 01.90000
 7   1011 STATION            30 12 30.51   w 90 43 37.08000
 7   1012 STATION            30 21 49.15   w 90 45 23.98000
 7   1013 STATION            29 59 45.81   w 90 59 14.06000
 7   1014 STATION            30 13 29.75   w 90 59 33.45000
 7   1015 STATION            30 23 53.10   w 90 58 47.00000
 7   1016 STATION            29 58 50.08   w 91 15  6.39000
 7   1017 STATION            30 12 45.48   w 91 15  2.23000
*
* GEOIDAL DATA RECORDS
*
 9300NO SUCH MOUNTAIN             0  0  .00010  0  0  .00010   0.000
 93001001 STATION               0  0  .00010  0  0  .00010  10.200
 93001002 STATION               0  0  .00010  0  0  .00010  10.100
```

```
93001003 STATION                0 0 .00010 0 0 .00010 10.000
93001004 STATION                0 0 .00010 0 0 .00010  9.800
93001005 STATION                0 0 .00010 0 0 .00010  9.800
93001006 STATION                0 0 .00010 0 0 .00010  9.800
93001007 STATION                0 0 .00010 0 0 .00010  9.600
93001008 STATION                0 0 .00010 0 0 .00010  9.500
93001009 STATION                0 0 .00010 0 0 .00010  9.400
93001010 STATION                0 0 .00010 0 0 .00010  9.600
93001011 STATION                0 0 .00010 0 0 .00010  9.200
93001012 STATION                0 0 .00010 0 0 .00010  9.400
93001013 STATION                0 0 .00010 0 0 .00010  9.300
93001014 STATION                0 0 .00010 0 0 .00010  9.100
93001015 STATION                0 0 .00010 0 0 .00010  9.000
93001016 STATION                0 0 .00010 0 0 .00010  8.900
93001017 STATION                0 0 .00010 0 0 .00010  9.100
93001018 STATION                0 0 .00010 0 0 .00010  9.000
*
* AUXILIARY PARAMETER DECLARATIONS
*
94disDISSCAL          SCAL
943dcDOPXF            ROTX ROTY ROTZ SCAL
943ddGPSXF            ROTX ROTY ROTZ SCAL
*
* DIRECTION OBSERVATIONS
*
10
512 0.7
 12 1001 STATION      1002 STATION          0  0  0.00000  .70000
 12 1001 STATION      NO SUCH MOUNTAIN     36 22 19.00000  .70000
 12 1001 STATION      1004 STATION        273 39 55.81000  .70000
 12 1001 STATION      1005 STATION        307 35 44.15000  .70000
 12 1002 STATION      1001 STATION          0  0  0.00000  .70000
 12 1002 STATION      1005 STATION         89  7 46.95000  .70000
 12 1002 STATION      1003 STATION        168 56 24.90000  .70000
 12 1002 STATION      NO SUCH MOUNTAIN    324 56  2.00000  .70000
 12 1003 STATION      1002 STATION          0  0  0.00000  .70000
 12 1003 STATION      1005 STATION         47 52 52.20000  .70000
 12 1003 STATION      1006 STATION         87 51 43.81000  .70000
 12 1004 STATION      1001 STATION          0  0  0.00000  .70000
 12 1004 STATION      1007 STATION        165  1 48.21000  .70000
 12 1004 STATION      1005 STATION        257 41  6.38000  .70000
 12 1004 STATION      NO SUCH MOUNTAIN    340 48 40.00000  .70000
 12 1005 STATION      1001 STATION          0  0  0.00000  .70000
 12 1005 STATION      1004 STATION         43 45 22.67000  .70000
 12 1005 STATION      1007 STATION         97 32  3.44000  .70000
 12 1005 STATION      1008 STATION        137  6 52.31000  .70000
 12 1005 STATION      1009 STATION        180  0 39.45000  .70000
 12 1005 STATION      1006 STATION        223 47 36.15000  .70000
 12 1005 STATION      1003 STATION        269 13 30.41000  .70000
 12 1005 STATION      1002 STATION        321 32  4.11000  .70000
 12 1006 STATION      1003 STATION          0  0  0.00000  .70000
 12 1006 STATION      1005 STATION         94 35 13.50000  .70000
 12 1006 STATION      1009 STATION        191 24 35.41000  .70000
 12 1007 STATION      1004 STATION          0  0  0.00000  .70000
```

```
12  1007 STATION     1010 STATION     191 56 23.05000   .70000
12  1007 STATION     1011 STATION     235  3 54.67000   .70000
12  1007 STATION     1008 STATION     278 17 28.81000   .70000
12  1007 STATION     1005 STATION     326 26  1.39000   .70000
12  1008 STATION     1005 STATION       0  0  0.00000   .70000
12  1008 STATION     1007 STATION      92 16 37.74000   .70000
12  1008 STATION     1011 STATION     184 45 23.59000   .70000
12  1008 STATION     1009 STATION     269 24 40.25000   .70000
12  1009 STATION     1005 STATION       0  0  0.00000   .70000
12  1009 STATION     1008 STATION      46 30 55.20000   .70000
12  1009 STATION     1011 STATION      90 40 53.58000   .70000
12  1009 STATION     1012 STATION     127 56   .10000   .70000
12  1009 STATION     1006 STATION     320 36 12.55000   .70000
12  1010 STATION     1007 STATION       0  0  0.00000   .70000
12  1010 STATION     1013 STATION     170 12 55.49000   .70000
12  1010 STATION     1011 STATION     262 32 42.46000   .70000
12  1011 STATION     1007 STATION       0  0  0.00000   .70000
12  1011 STATION     1010 STATION      39 25 12.88000   .70000
12  1011 STATION     1013 STATION      89  4 44.90000   .70000
12  1011 STATION     1014 STATION     136 42 14.89000   .70000
12  1011 STATION     1015 STATION     173 27 34.71000   .70000
12  1011 STATION     1012 STATION     212 33 47.39000   .70000
12  1011 STATION     1009 STATION     264 31 31.23000   .70000
12  1011 STATION     1008 STATION     315 42 19.42000   .70000
12  1012 STATION     1009 STATION       0  0  0.00000   .70000
12  1012 STATION     1011 STATION      90 47  7.63000   .70000
12  1012 STATION     1015 STATION     199 30 15.18000   .70000
12  1013 STATION     1010 STATION       0  0  0.00000   .70000
12  1013 STATION     1016 STATION     181 29 32.08000   .70000
12  1013 STATION     1017 STATION     228 28 33.78000   .70000
12  1013 STATION     1014 STATION     273 33 42.81000   .70000
12  1013 STATION     1011 STATION     321 59 17.07000   .70000
12  1014 STATION     1011 STATION       0  0  0.00000   .70000
12  1014 STATION     1013 STATION      83 56 57.60000   .70000
12  1014 STATION     1017 STATION     172 24 26.66000   .70000
12  1014 STATION     1015 STATION     269 40 59.47000   .70000
12  1015 STATION     1011 STATION       0  0  0.00000   .70000
12  1015 STATION     1014 STATION      52 55 42.06000   .70000
12  1015 STATION     1017 STATION     100 35   .28000   .70000
12  1015 STATION     1018 STATION     144 36 30.28000   .70000
12  1015 STATION     1012 STATION     327 49 16.76000   .70000
12  1016 STATION     1013 STATION       0  0  0.00000   .70000
12  1016 STATION     1017 STATION     273 49 41.47000   .70000
12  1017 STATION     1013 STATION       0  0  0.00000   .70000
12  1017 STATION     1016 STATION      46 50 42.45000   .70000
12  1017 STATION     1018 STATION     231 49 22.16000   .70000
12  1017 STATION     1015 STATION     278 28 27.94000   .70000
12  1017 STATION     1014 STATION     313 32 37.58000   .70000
12  1018 STATION     1015 STATION       0  0  0.00000   .70000
12  1018 STATION     1017 STATION      89 19 25.16000   .70000
*
* DISTANCE OBSERVATIONS
*
```

```
52Y     0.0   2.0          .0  .0                   DISSCAL
 2Y  1003 STATION          1002 STATION               27464.39500  5.492879
 2Y  1006 STATION          1005 STATION               22020.80400  4.404168
 2Y  1009 STATION          1008 STATION               23448.91400  4.689782
 2Y  1012 STATION          1011 STATION               18140.43400  3.628086
 2Y  1015 STATION          1014 STATION               19305.54400  3.861108
 2Y  1018 STATION          1017 STATION               23165.24100  4.633048
*
* AZIMUTH OBSERVATIONS
*
53B  1.0
 3B  1005 STATION          1004 STATION           175 51 33.33000   1.00000
 3B  1010 STATION          1014 STATION           311  5 55.59000   1.00000
 3B  1015 STATION          1014 STATION           184 17 23.14000   1.00000
*
* DOPPLER POSITION OBSERVATIONS
*
953dcDOPPLER POSITION OBSERVATIONS
92   1001 STATION                   -55.64 -5530068.70   3171200.3400
92   1003 STATION                 -1497.17 -5506232.99   3212211.510
92   1005 STATION                -24110.43 -5519043.83   3190045.920
92   1009 STATION                -49612.74 -5507343.03   3210029.190
92   1010 STATION                -69214.24 -5528784.75   3172735.750
92   1014 STATION                -95655.96 -5516780.82   3192779.730
92   1016 STATION               -120827.22 -5529963.78   3169482.250
92   1018 STATION               -118126.76 -5505349.97   3211543.350
943dcDOPXF
97POVDIAGONAL
98 0.01           0.01           0.01
98 0.01           0.01           0.01
98 0.01           0.01           0.01
98 0.01           0.01           0.01
98 0.01           0.01           0.01
98 0.01           0.01           0.01
98 0.01           0.01           0.01
98 0.01           0.01           0.01
*
*  VLBI POSITION DIFFERENCE OBSERVATIONS
*
913ddVLBI POSITION DIFFERENCE OBSERVATIONS
92   1001 STATION                  0.00     -5530065.28    3171180.470
92   1009 STATION              -49557.16    -5507340.00    3210009.750
92   1010 STATION              -69158.69    -5528781.74    3172716.390
92   1018 STATION             -118070.97    -5505347.69    3211524.130
97PDVUPPER
98  0.25E-4         0.20E-5         0.0
98  0.0          -0.40E-5         0.0
98  0.0           0.0          0.0
98  0.16E-4         0.0          -0.12E-5
98 -0.16E-5         0.0           0.0
98  0.0          -0.40E-5
98  0.90E-5         0.0           0.0
98  0.0           0.15E-5         0.15E-5
98  0.0
```

```
98  0.90E-5        0.0          0.0
98  0.15E-5        0.0          0.0
98  0.16E-4       -0.20E-5       -0.40E-5
98  0.0          0.0
98  0.25E-4        0.0          0.0
98  0.0
98  0.25E-4        0.0          0.0
98  0.25E-4        0.25E-5
98  0.25E-4
*
* GPS POSITION DIFFERENCE OBSERVATIONS
*
913ddGPS POSITION DIFFERENCE OBSERVATIONS
92   1010 STATION              -69251.20     -5528758.12  3172774.76
92   1013 STATION              -95267.38     -5529349.94  3170743.46
92   1014 STATION              -95692.56     -5516753.96  3192818.800
92   1016 STATION             -120864.08     -5529936.89  3169521.400
943ddGPSXF
97PDVDIAGONAL
98 1.0E-4          1.E-4          2.25E-4
98 1.0E-4          0.64E-4         1.0E-4
98 1.0E-4          1.0E-4         1.0E-4
99
```

Table 7.3 NETAN listing of reliability analysis results for simulated 3D HOACS network.

```
-------------------------------------------
NETAN:  Network Analysis (Version 21 Nov 90)
Network Strength Analysis
-------------------------------------------

Piece-Wise Linear Approximation -- Connected Stations

Input network data file : []
  (NETAN) TEST HOACS GHOST  TERRESTRIAL DATA


Station     Name
          Lat (DMS), Long (DMS), Ht (m)
         Strength in Rotation: Lat/Lon, Lat/Ht, Lon/Ht (rad)
            Obs # and Type
         Strength in Shear: Lat/Lon, Lat/Ht, Lon/Ht (strain)
            Obs # and Type
         Strength in Scale: (strain)
            Obs # and Type

  1    NO SUCH M    MOUNTAIN
         30  4 59.849800   -89 54 59.736034       3500.000000
        -0.8026581412E-05   0.0000000000E+00   0.0000000000E+00
              8 dir           0              0
         0.1091513284E-04   0.0000000000E+00   0.0000000000E+00
              8 dir           0              0
         0.9239484003E-05
             114 pos

  2    1001 STAT    TION
         29 59 59.851957   -89 59 59.734301        1966.254360
        -0.2388609726E-05   0.0000000000E+00   0.0000000000E+00
             111 pos           0              0
         0.3112551925E-05   0.0000000000E+00   0.0000000000E+00
             114 pos           0              0
         0.3395127742E-05
              90 dpos

  3    1002 STAT    TION
         30 10 53.849676   -89 58 58.732088        2174.420000
        -0.5816950042E-05   0.0000000000E+00   0.0000000000E+00
              8 dir           0              0
         0.7489362239E-05   0.0000000000E+00   0.0000000000E+00
              8 dir           0              0
         0.9625842198E-05
              90 dpos

  4    1003 STAT    TION
         30 25 39.851202   -90  0 53.729260       2006.840615
        -0.5517790657E-05   0.0000000000E+00   0.0000000000E+00
```

```
        111 pos          0             0
  0.4320969610E-05   0.0000000000E+00   0.0000000000E+00
        90 dpos          0             0
  0.1044322603E-04
        90 dpos

5   1004 STAT   TION
    30  1 46.853908   -90 14  8.733804      2024.990000
  -0.4193263814E-05   0.0000000000E+00   0.0000000000E+00
        111 pos          0             0
  0.5046679535E-05   0.0000000000E+00   0.0000000000E+00
        8 dir          0             0
  0.4409285201E-05
        90 dpos

6   1005 STAT   TION
    30 11 47.852079   -90 14 58.732385      1924.276268
  -0.4299814151E-05   0.0000000000E+00   0.0000000000E+00
        111 pos          0             0
  0.2644233588E-05   0.0000000000E+00   0.0000000000E+00
        111 pos          0             0
  0.5075764374E-05
        90 dpos

7   1006 STAT   TION
    30 23 40.852787   -90 15 57.734057      2212.980000
  -0.4911522032E-05   0.0000000000E+00   0.0000000000E+00
        111 pos          0             0
  0.5689289749E-05   0.0000000000E+00   0.0000000000E+00
        90 dpos          0             0
  0.6913426577E-05
        90 dpos

8   1007 STAT   TION
    30  0  1.854932   -90 30 51.730790      1946.620000
  -0.1625908368E-05   0.0000000000E+00   0.0000000000E+00
        44 dir          0             0
  0.2272408046E-05   0.0000000000E+00   0.0000000000E+00
        31 dir          0             0
  -0.6314001371E-05
        114 pos

9   1008 STAT   TION
    30 11 35.852757   -90 30 32.729836      2085.430000
  -0.2329687269E-05   0.0000000000E+00   0.0000000000E+00
        44 dir          0             0
  0.2080895605E-05   0.0000000000E+00   0.0000000000E+00
        96 dpos          0             0
  -0.6123746780E-05
        114 pos

10  1009 STAT   TION
    30 24 16.851410   -90 30 55.730357      2051.734562
  -0.4319115962E-05   0.0000000000E+00   0.0000000000E+00
```

```
          111 pos        0            0
     0.3635786501E-05   0.0000000000E+00   0.0000000000E+00
          114 pos        0            0
     0.4993779365E-05
          111 pos

 11   1010 STAT    TION
      30  0 56.853268   -90 42 59.731058     1997.382402
     -0.4370081402E-05   0.0000000000E+00   0.0000000000E+00
          117 pos        0            0
     0.7321804042E-05   0.0000000000E+00   0.0000000000E+00
          123 pos        0            0
     0.5585010897E-05
          123 pos

 12   1011 STAT    TION
      30 12 20.852495   -90 43 34.725644     1964.020000
     -0.4090951484E-05   0.0000000000E+00   0.0000000000E+00
          117 pos        0            0
     0.2837360333E-05   0.0000000000E+00   0.0000000000E+00
          123 pos        0            0
     0.4395271148E-05
          111 pos

 13   1012 STAT    TION
      30 22  1.852968   -90 45 25.723984     1883.560000
     -0.3485735196E-05   0.0000000000E+00   0.0000000000E+00
          117 pos        0            0
     0.6204770709E-05   0.0000000000E+00   0.0000000000E+00
          111 pos        0            0
     0.5532237335E-05
          111 pos

 14   1013 STAT    TION
      29 59 43.854453   -90 59  9.729649     1829.087482
     -0.7154654314E-05   0.0000000000E+00   0.0000000000E+00
          117 pos        0            0
     0.7849933001E-05   0.0000000000E+00   0.0000000000E+00
          123 pos        0            0
     0.5357576345E-05
          123 pos

 15   1014 STAT    TION
      30 13 28.853227   -90 59 33.729039     2015.332708
     -0.7117499833E-05   0.0000000000E+00   0.0000000000E+00
          117 pos        0            0
     0.3786634816E-05   0.0000000000E+00   0.0000000000E+00
          123 pos        0            0
     -0.3498957363E-05
          105 dpos

 16   1015 STAT    TION
      30 23 53.852800   -90 58 39.727600     1959.240000
     -0.8411430126E-05   0.0000000000E+00   0.0000000000E+00
```

```
        117 pos          0              0
     0.3517979138E-05   0.0000000000E+00   0.0000000000E+00
        114 pos          0              0
     0.3743625715E-05
        90 dpos

17   1016 STAT   TION
      29 58 51.854512   -91 15  3.728499      2159.675769
     -0.8032074231E-05   0.0000000000E+00   0.0000000000E+00
        117 pos          0              0
      0.1062148367E-04   0.0000000000E+00   0.0000000000E+00
        123 pos          0              0
     -0.7689695578E-05
        105 dpos

18   1017 STAT   TION
      30 12 45.855024   -91 15  0.725979      2080.350000
     -0.9132876044E-05   0.0000000000E+00   0.0000000000E+00
        117 pos          0              0
      0.4900893724E-05   0.0000000000E+00   0.0000000000E+00
        126 pos          0              0
     -0.5868983027E-05
        126 pos

19   1018 STAT   TION
      30 25 14.852635   -91 13 42.726788      1999.590054
     -0.6260417822E-05   0.0000000000E+00   0.0000000000E+00
        117 pos          0              0
      0.3538720212E-05   0.0000000000E+00   0.0000000000E+00
        126 pos          0              0
      0.3535383204E-05
        90 dpos
```

7.3 Real 2D Network

A real 2D network was obtained from the Canadian Geodetic Survey for an area in Québec along the south shore of the St. Lawrence River. The network consists of a total of 58 stations. Only one point is fixed and a single azimuth observation is used to control the datum orientation. The network consists of the following observations:

- 125 distance observations with standard deviations ranging from 1 cm + 2 ppm to 6 cm + 6 ppm (most around 3 cm + 3 ppm)
- 307 direction observations with standard deviations ranging from 0.6 to 2.0 seconds (most around 0.7 seconds)
- 1 azimuth observation with a standard deviation of 0.8 seconds.

Figures 7.9 to 7.11 illustrate the locations of the stations and the different types of observations. Table 7.5 gives a listing of the input GHOST data file for this network.

The results from the NETAN robustness analysis are displayed in terms of robustness in rotation (local twisting), robustness in shear (local changes in configuration or shape), and robustness in scale in Figures 7.12, 7.13, and 7.14, respectively. The NETAN output listing for this analysis is given in Table 7.6. These robustness results are all based on α_0=5% and β_0=5% which gives a non-centrality parameter of $\sqrt{\lambda_0}$=3.61 (the standardized value of maximum undetectable error). Different α_0 and β_0 result in a different $\sqrt{\lambda_0}$ which only scales the magnitude of the strength parameters by the ratio of the non-centrality parameters. The plots are otherwise identical.

The results indicate that the weakest point in the network is at station HEMMING where the differential rotation is 33 μrad, shear is 39 ppm, and dilation is 21 ppm. These relatively large deformations, however, are actually a result of a weakness in the determination of the strain primitives rather than a weakness in the network itself. The points connected to this station are nearly collinear which makes the fitting of a plane surface to these points ill-conditioned for the determination of the strain elements. This problem with the determination of strain has already been pointed out by Craymer et al. [1989]. It may be possible to detect such ill-conditioning by computing the determinant of the matrix of normal equations for the determination of the strain elements. On the other hand, it would be more useful in general to compute standard deviations for the strain primitives and test them for statistical significance. Station HEMMING will therefore not be considered further in the following discussion of the results.

A number of points were found to be uniquely determined by a minimum number of observations. These observations were omitted from the robustness analysis since their redundancy numbers are zero resulting in infinitely large maximum undetectable errors. The robustness parameters for these points have been set to zero in the output listing. As discussed in the previous chapter, it is recommended to include these observations and points in the next version of NETAN by setting their maximum undetectable errors to a very large number (say, 0.33E33) so that they will show up as very large weaknesses in the contour plots of the robustness parameters.

The single azimuth controlling the datum orientation was also omitted from the analysis since its redundancy number was very small. This would give a very large maximum undetectable error that would overwhelm the robustness analysis. As discussed in the previous chapter, we also recommend that in a future version of NETAN these observations be included in the robustness analysis so that the weakness in datum definition would be evident as large robustness parameters in the contour plots.

The results indicate that the north part of the network is relatively weaker than the south part. Table 7.4 summarizes the range of values, the largest and smallest (absolute) values in magnitude, and the average and standard deviation for each robustness parameter. Note again that the average and standard deviations of these parameters are all of the same magnitude as the relative accuracy of the observations. Values less than about 10 ppm are therefore probably not very statistically significant. The only obviously common feature to most of these weak points is the lack of direction observations emanating from them. Although it seems reasonable that this may have a detrimental effect on differential rotation and shear, it is not clear why this would also cause relatively large dilations.

Table 7.4 Summary of robustness results for the real network.

	Robustness in rotation (μrad)	Robustness in shear (ppm)	Robustness in scale (ppm)
Maximum	16.2	38.7	22.5
Minimum	−19.8	0.6	−4.2
Largest absolute	−19.8	38.7	22.5
Smallest absolute	1.8	0.6	0.7

The robustness in rotation results are illustrated in Figure 7.12 and listed in Table 7.6. These results describe the ability of the network to resist local changes in orientation (twisting). The largest values are obtained for station STRATFORD (−19.8 μrad due to direction observation #228), station KINGSEY FALLS (16.2 μrad due to distance observation #412), station ADSTOCK (−12.3 μrad due to direction observation #151), station ARTHABASKA (11.9 μrad due to distance observation #412), and station ST ZEPHIRIN (−11.1 μrad due to distance observation #401). All these stations have relatively fewer observation ties connecting them to the rest of the network (generally only a few directions and distances at most). Note that these values are within first-order accuracy standards, however. The best (smallest absolute value) robustness in rotation is obtained for station ST ARMAND (1.8 μrad) which has many more observation ties (14 directions and 2 distances) than the weak points.

The robustness in shear results are given in Figure 7.13 and Table 7.6. These results describe the ability of the network to resist local deformations in configuration or shape. The largest values are obtained for station ST ZEPHIRIN (21.4 ppm due to distance observation #401), station STRATFORD (17.8 ppm due to direction observation #228), station ARTHABASKA (15.9 ppm due to distance observation #412), station VIANNEY (15.0 ppm due to distance observation #383), station KINGSEY FALLS (14.0 ppm due to distance observation #412), and station GILBERT (13.5 ppm due to position observation #123). All of these points have relatively fewer observation ties connecting them to the rest of the network. With the exception of ST ZEPHIRIN, these values are also within first-order accuracy limits. The best (smallest absolute value) robustness in shear is obtained for station HEREFORD (0.6 ppm) which has many more observation ties (16 directions and 4 distances) than the weak points.

The robustness in scale results are given in Figure 7.14 and Table 7.6. These results describe the ability of the network to resist local deformations in scale (dilation). The largest values are obtained for station VIANNEY (22.5 ppm due to distance observation #383), station ARTHABASKA (18.0 ppm due to distance observation #412), station ST ZEPHIRIN (17.0 ppm due to distance observation #401), station STRATFORD (15.4 ppm due to distance observation #422), station ADSTOCK (12.9 ppm due to distance observation #395), and station KINGSEY FALLS (11.6 ppm due to direction observation #244). All of these points have relatively fewer observation ties connecting them to the rest of the network. With the exception of VIANNEY, these values are also within first-order accuracy limits. The best (smallest absolute value) robustness in scale is obtained for station SHERBROOKE (0.7 ppm) which has many more observation ties (18 directions and 9 distances) than the weak points.

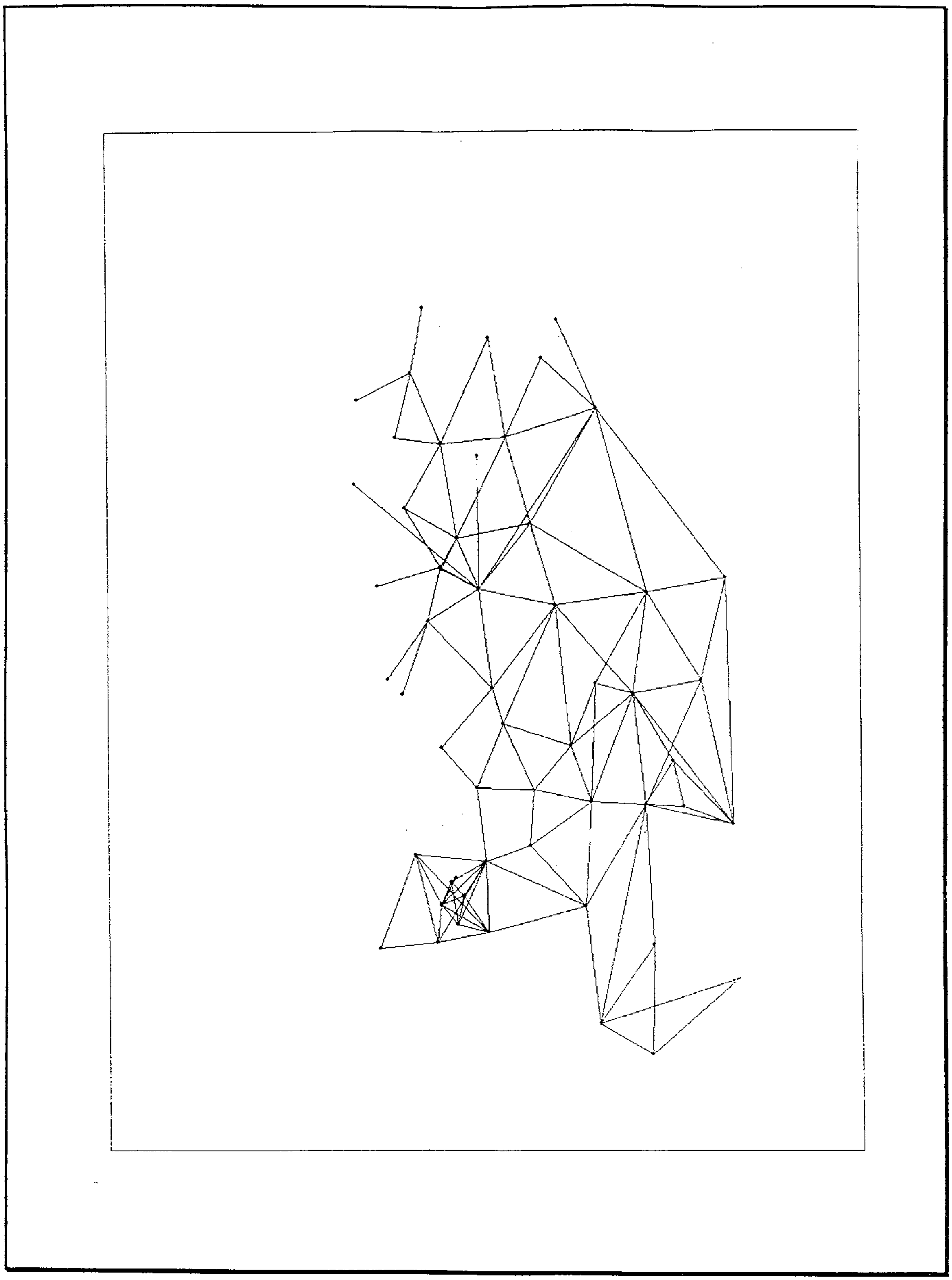

Figure 7.9 Distance observations for real 2D network.

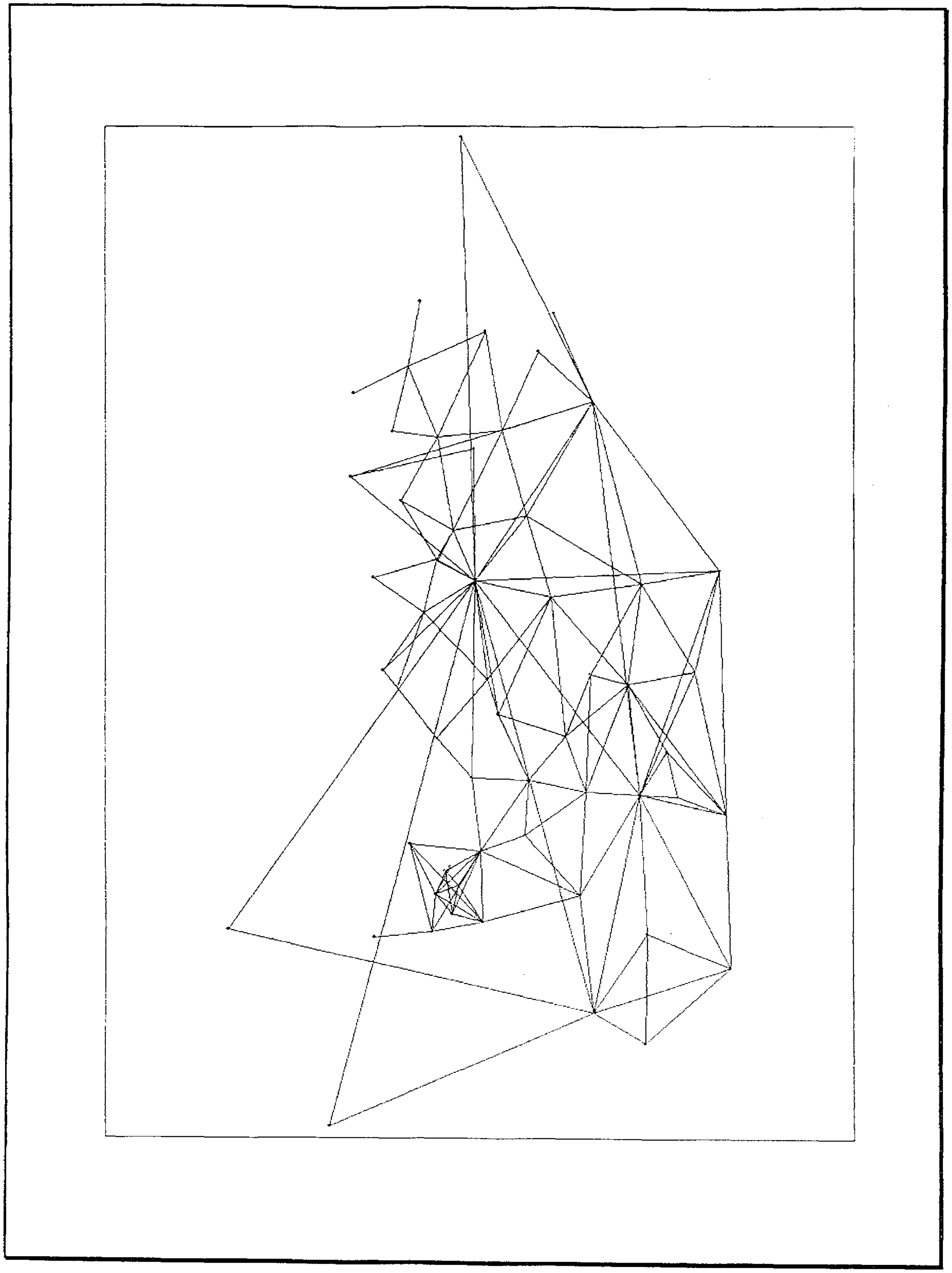

Figure 7.10 Direction observations for real 2D network.

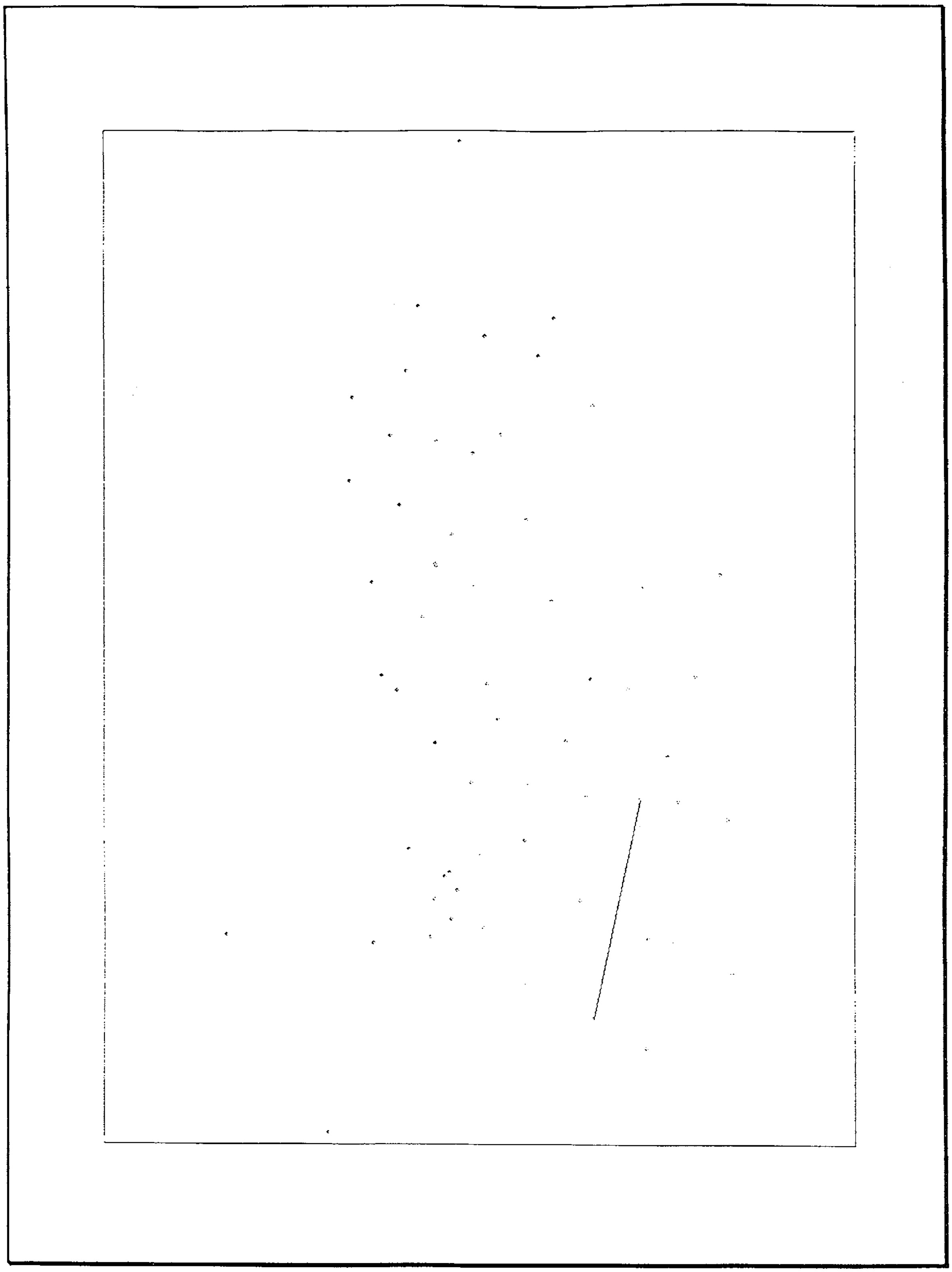

Figure 7.11 Azimuth observation for real 2D network.

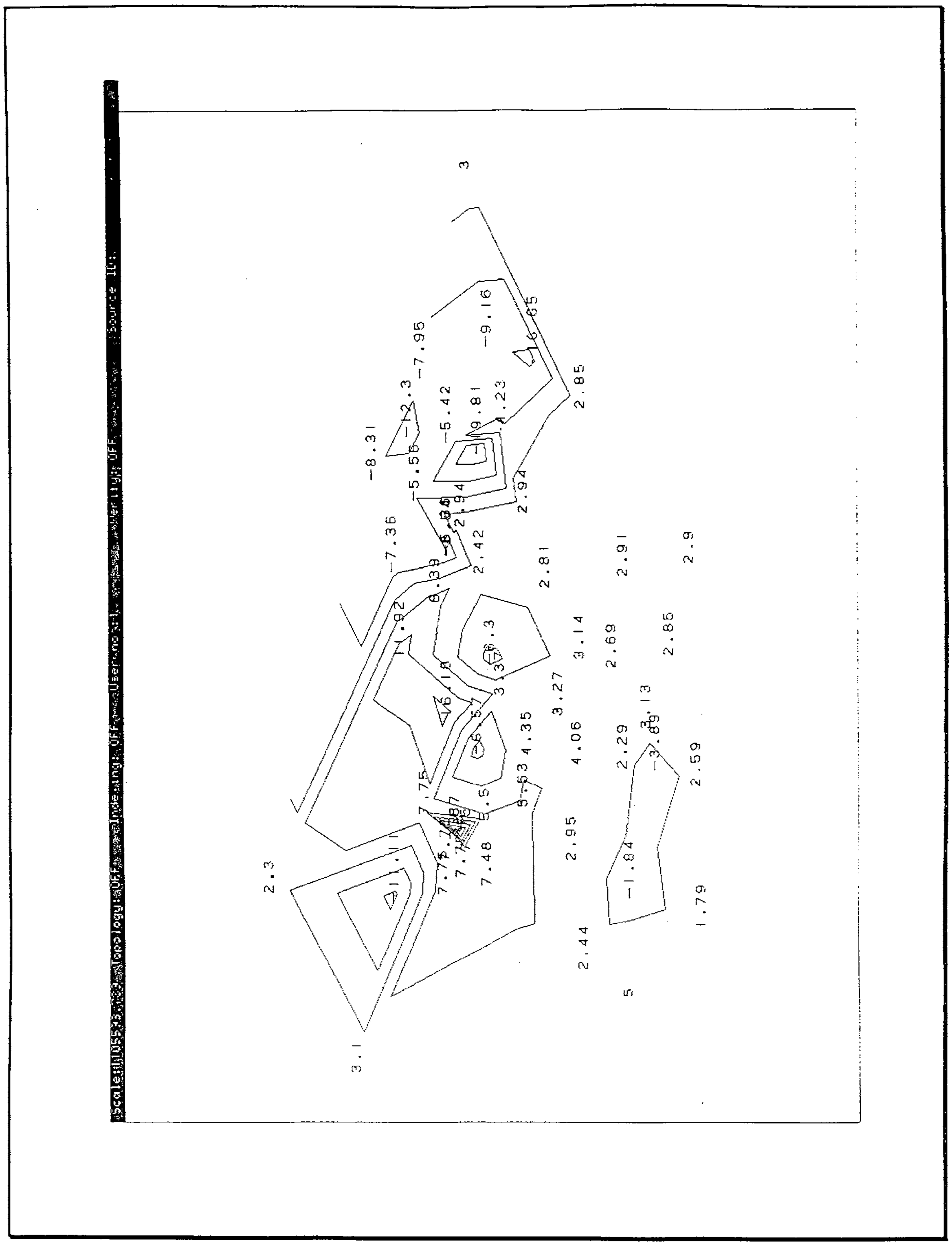

Figure 7.12 Robustness in rotation for real 2D network.

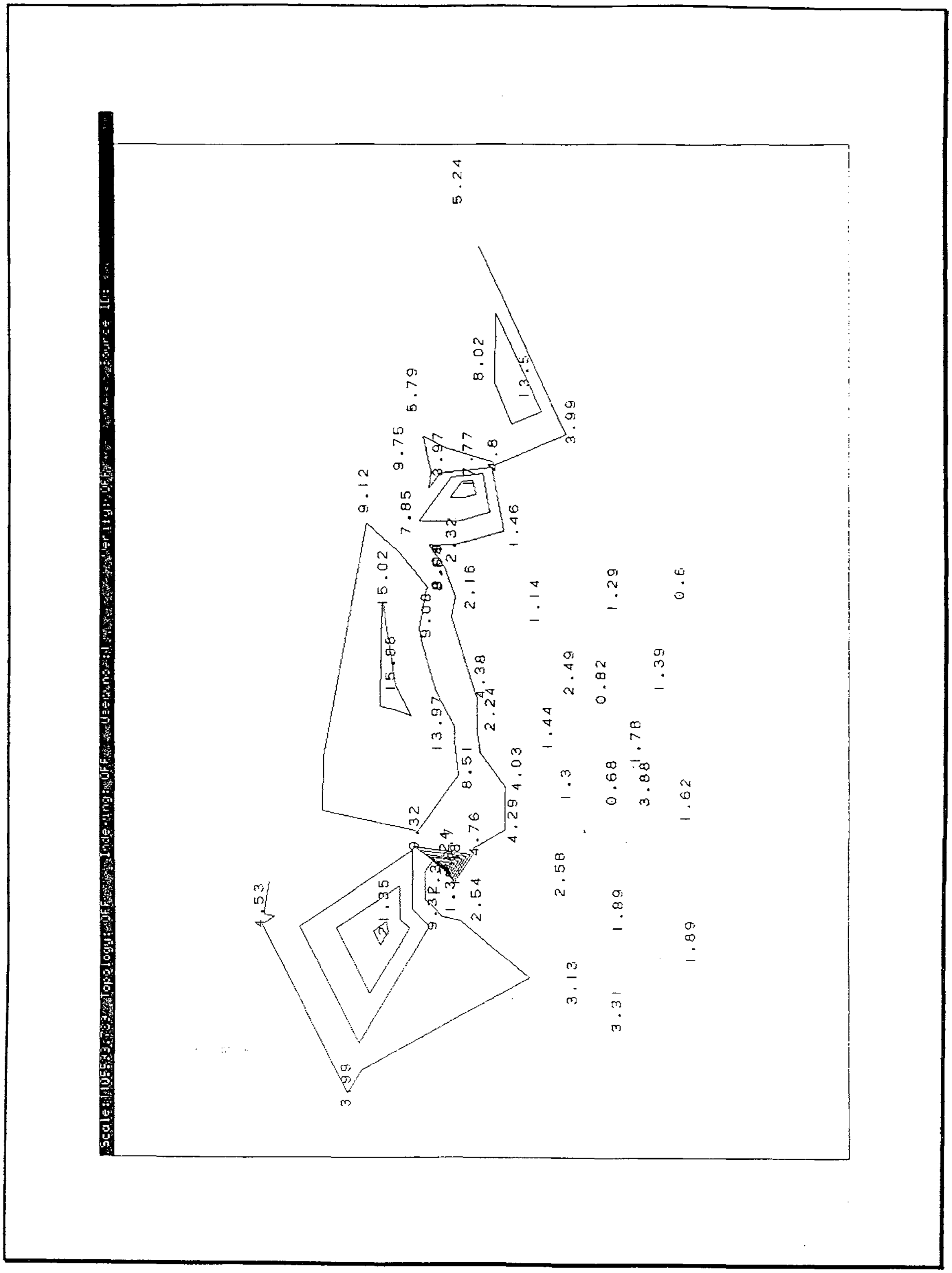

Figure 7.13 Robustness in shear for real 2D network.

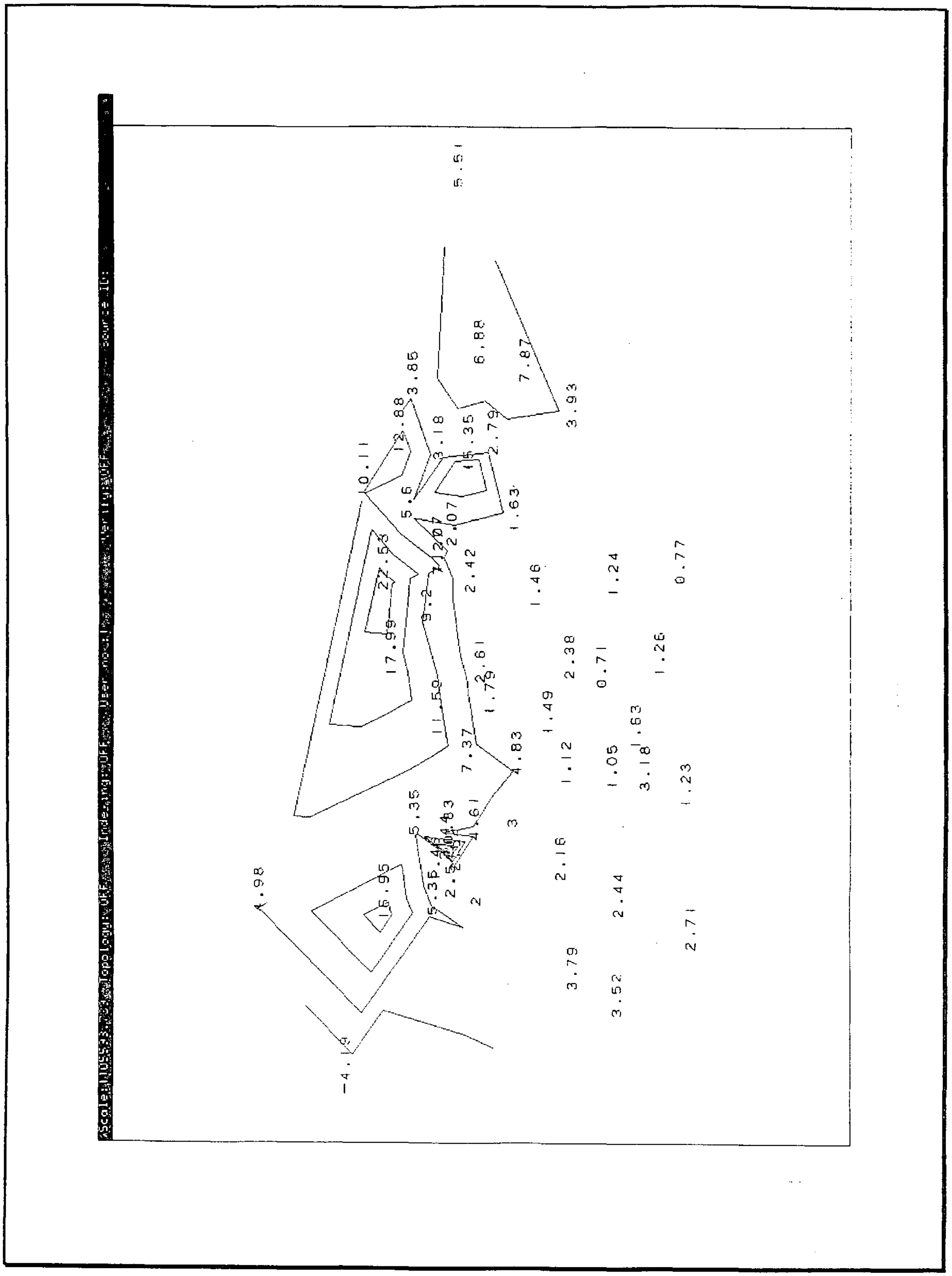

Figure 7.14 Robustness in scale for real 2D network.

Table 7.5 GHOST input data file for real 2D network.

```
Real 2D Network, Geodetic Survey of Canada. Sigma records commented out.
 1 3  1  1 1 1 1
 4  09206  ORFORD            N45 1843.080825W 72 1430.207541 851.9000
10
 5  08200  ST ARMAND          N45  246.875283W 72 4420.977882 711.7200
 5  09201  YAMASKA            N45 2645.375984W 72 52 8.649835 421.7800
 4  652401  FARNHAM           N45 1743.910275W 72 5719.624866  77.9800
 4  652402  BROMONT           N45 1721.079434W 72 3816.142978 552.8000
 4  69K4238 DAIGLE        1  QLF N45 29 9.969288W 72 3138.846985 276.3000
 5  712051  ST MAJORIQUE   1  QLF N45 5458.877438W 72 38 1.278776  82.9280
 4  712056  WICKHAM       1  QLF N45 46 7.568544W 72 3621.113477 106.8230
 4  08207  OWLS HEAD          N45  345.156706W 72 1752.895781 749.9000
 5  09202  DUSABLE        1  274 N46 1237.122695W 73 1159.764446 135.9410
 5  09204  CARMEL         1  173 N46 2958.646202W 72 3739.182006 188.6410
 5  09205  HAM               N45 4728.148868W 71 38  .735368 711.7000
 4  09207  HEREFORD           N45  457.209129W 71 36 3.594684 872.1000
 4SBF 09208  MEGANTIC          N45 2651.257597W 71  713.0032151085.7875
 5  09209  THETFORD           N46  848.515359W 71 2011.406728 694.0700
 5  09210  LINIERE           N45 4945.092500W 70 2220.286423 776.0900
 5  09216  ARTHABASKA      1  233 N46  314.110312W 71 5316.763943 350.2910
 5  14200  STRATFORD      2  233 N45 4739.752238W 71 1520.012822 436.9000
 5  65K0335 CROIX            N45 3346.481649W 70 5222.732410 490.8600
 5  66KP115 CARIBOU E-15   1  164 N46  015.967744W 71 2410.058178 556.7000
 4  68K2071 SHERBROOKE         N45 2046.261393W 71 5533.821339 440.3000
 4  68K2073 BEAUVOIR      1  164 N45 2717.104197W 71 5353.741152 307.9000
 5  692009  HONORE            N45 5653.160925W 70 5016.132369 474.3400
 5  692010  GRELOTS       1  172 N45 59 2.193228W 71  121.630665 408.7890
 5  692011  BROUGHTON     1  172 N46  817.581129W 71  549.572672 608.5880
 5  692012  ADSTOCK       1  164 N46  146.972910W 71 1218.355831 713.1880
 4  69K4239 DUSSAULT      1  164 N45 28 4.320930W 72 1351.756838 430.8000
 4  69K4240 GALLUP HILL   1  164 N45 38 6.700659W 72 1157.248631 348.2000
 4  69K4241 SOUTH DURHAM   1  164 N45 3848.371625W 72 2126.889288 208.8100
 4  69K4242 PINNACLE      1  QLF N45 4321.445932W 72  041.591747 416.4000
 4  69K4243 LAROCHELLE    1  164 N45 3143.227349W 72  423.551476 333.1000
 4  69K4346 CHARLES       1  163 N45 5234.275759W 72 2739.825309  93.3900
 4  69K4348 LEMAIRE       1  163 N45 5123.415278W 72 3452.384059  93.5300
 4  69K4349 BREBOEUF      1  163 N45 5020.787836W 72 2959.667450  89.2000
 4  69K4350 HEMMING       2  163 N45 5146.549886W 72 27  .762848 115.3700
 4  70K4244 MAGOG              N45 1357.376894W 72  7 2.330626 345.0000
 4  70K4245 AUSTIN             N45 12 7.761103W 72 1445.003137 318.3000
 4  70K4631 HATLEY             N45  9 8.355020W 71 5318.368308 423.0000
 4  70K4632 MARTIN             N45 1823.800369W 71 3831.348936 423.1000
 4  70K4633 CHAPMAN       1  164 N45 3416.980871W 71 4044.156151 658.4000
 4  70K4634 ASBESTOS      1  QLF N45 4516.870156W 71 5442.774960 338.1000
 4  70K4635 WEEDON             N45 3832.841031W 71 2642.205826 413.6000
 5  70K4637 GILBERT            N45 3621.022836W 70 5844.386984 570.8000
 5  70K4638 COULOMBE      1  QLF N45 5112.412896W 71 2919.000750 465.8000
 4  70K4639 MOISAN       4  164 N45 5356.869207W 71 3435.946548 597.5000
 5  712050  BON CONSEIL    1  174 N45 5840.686135W 72 2258.068549 121.2210
 5  712053  ST ZEPHIRIN    1  174 N46  447.587703W 72 39 2.801747  55.2430
 4  712055  MALLARD       1  QLF N45 4628.674715W 72 24 5.669217 171.4480
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4   712057  DRUMMOND       1   174 N45 5416.673859W 72 3135.417580  93.6700
5   71K6154 STORNOWAY            N45 4245.378012W 71 1213.496977 510.0000
5   71K6155 STE PRAXEDE    1   164 N45 5351.210905W 71 1323.129308 391.4000
5   71K6156 SEBASTIEN           N45 4536.557799W 70 5519.545890 825.7000
5   71K6159 LAPOINTE       1   QLF N45 54 6.371447W 71 3414.951193 627.8000
5   71K6165 VIANNEY        1   QLF N46  452.995542W 71 3730.212463 592.3000
5   72K7455 VICTORIAVILLE  1   QLF N46  041.609238W 71 5546.742907 274.7530
5   72K7457 SEVIGNY        1   QLF N45 5613.677425W 71 4317.715169 528.2000
5   72K7462 KINGSEY FALLS  1   QLF N45 54 5.083744W 72  440.296902 154.5000
5   72K7463 ST FELIX       1   QLF N45 4758.866518W 72 1128.912506 209.6000
7 36 09202  DUSABLE         274 N46 12 36.48   W 73 12 01.40    135.9
7 71 09206  ORFORD          233 N45 18 39.31   W 72 14 30.93    838.
7 72 09208  MEGANTIC        233 N45 26 52.92   W 71 07 14.60    1082.
7 38 09216  ARTHABASKA      233 N46 03 26.01   W 71 53 35.16    350.291
7 72 692009 HONORE          164 N45 56 57.00   W 70 50 18.42    474.3
7 72 69K4241 SOUTH DURHAM   163 N45 38 53.69   W 72 21 41.72    206.9
7 72 712057 DRUMMOND        174 N45 54 25.52   W 72 31 50.58    93.7
9   08200   ST ARMAND     MAIN1.3    4.33     -10.02   -28.38
9   08207   OWLS HEAD     MAIN1.3   -1.11       1.25   -27.64
9   09201   YAMASKA       MAIN1.3    2.30      -8.89   -29.49
9   09202   DUSABLE       GEM10B1    1.22      -4.18   -31.16
9   09204   CARMEL        GEM10B1    1.45      -4.06   -30.02
9   09205   HAM           MAIN1.3   -2.39      -2.41   -27.72
9   09206   ORFORD        MAIN1.4   -3.12       -.48   -27.99
9   09207   HEREFORD      MAIN1.3    -.74      -5.85   -26.57
9   09208   MEGANTIC      MAIN1.3    3.21      -3.41   -26.32
9   09209   THETFORD      MAIN1.3    4.08       -.49   -27.50
9   09210   LINIERE       MAIN1.3    5.49      -8.74   -25.80
9   09216   ARTHABASKA    GEM10B2    1.05      -3.77   -28.56
9   14200   STRATFORD     GEM10B2     .73      -3.46   -27.03
9   652401  FARNHAM       MAIN1.3    1.91      -6.72   -29.28
9   652402  BROMONT       MAIN1.3    1.22     -12.62   -28.71
9   65K0335 CROIX         MAIN1.3    2.05      -4.21   -26.14
9   66KP115 CARIBOU E-15  GEM10B2     .97      -3.56   -27.54
9   68K2073 BEAUVOIR      GEM10B2     .40      -3.67   -27.60
9   692009  HONORE        GEM10B2     .87      -3.28   -26.57
9   692010  GRELOTS       MAIN1.3     .98        .87   -26.88
9   692011  BROUGHTON     GEM10B2    1.09      -3.44   -27.09
9   692012  ADSTOCK       MAIN1.3   -1.44       3.70   -27.21
9   68K2071 SHERBROOKE    GEM10B2     .27      -3.65   -27.45
9   69K4238 DAIGLE        GEM10B2     .50      -3.90   -28.95
9   69K4239 DUSSAULT      MAIN1.3   -1.05        .29   -28.30
9   69K4240 GALLUP HILL   GEM10B1     .63      -3.82   -28.61
9   69K4241 SOUTH DURHAM  GEM10B2     .66      -3.88   -28.97
9   69K4242 PINNACLE      GEM10B2     .71      -3.77   -28.39
9   69K4243 LAROCHELLE    GEM10B2     .50      -3.75   -28.11
9   69K4346 CHARLES       GEM10B1     .90      -3.95   -29.62
9   69K4348 LEMAIRE       GEM10B0     .89      -3.99   -29.83
9   69K4349 BREBOEUF      GEM10B0     .87      -3.96   -29.64
9   69K4350 HEMMING       GEM10B1     .89      -3.95   -29.57
9   70K4244 MAGOG         MAIN1.4   -2.76       -.19   -27.60
9   70K4245 AUSTIN        MAIN1.4   -2.62        .52   -27.78
9   70K4631 HATLEY        GEM10B2     .04      -3.59   -27.06
9   70K4632 MARTIN        GEM10B2     .19      -3.52   -26.88
```

```
 9   70K4633 CHAPMAN        MAIN1.3     -3.42        -1.45   -27.41
 9   70K4634 ASBESTOS       GEM10B2       .74        -3.74   -28.23
 9   70K4635 WEEDON         GEM10B2      .57        -3.51   -27.12
 9   70K4637 GILBERT        MAIN1.3     1.97       -2.60   -26.34
 9   70K4638 COULOMBE       MAIN1.3     -3.47         .35   -27.53
 9   70K4639 MOISAN         MAIN1.4    -3.03       -1.74   -27.76
 9   712050  BON CONSEIL    GEM10B0      1.00        -3.94   -29.59
 9   712051  ST MAJORIQUE   GEM10B0       .95        -4.01   -30.02
 9   712053  ST ZEPHIRIN    GEM10B0     1.10        -4.04   -30.22
 9   712055  MALLARD        GEM10B0      .80        -3.92   -29.32
 9   712056  WICKHAM        GEM10B0      .80        -3.98   -29.72
 9   712057  DRUMMOND       GEM10B0       .93        -3.98   -29.79
 9   71K6154 STORNOWAY      GEM10B2       .63        -3.42   -26.83
 9   71K6155 STE PRAXEDE    MAIN1.3     -1.41         1.24   -27.11
 9   71K6156 SEBASTIEN      MAIN1.3     2.48        -2.42   -26.47
 9   71K6159 LAPOINTE       MAIN1.4    -2.99        -1.67   -27.76
 9   71K6165 VIANNEY        MAIN1.3     5.99       -4.28   -28.02
 9   72K7455 VICTORIAVILLE  GEM10B1      1.01        -3.79   -28.62
 9   72K7457 SEVIGNY        GEM10B2      .92        -3.69   -28.10
 9   72K7462 KINGSEY FALLS  GEM10B1       .90        -3.83   -28.83
 9   72K7463 ST FELIX       GEM10B1      .81        -3.85   -28.92
*
* Observations
*
 40
* 51F16   0.85  0.0  0.01969  .20  .20G-213    QLF
 1   69K4238 DAIGLE        712056  WICKHAM        0  0  0.00000    .850
 1   69K4238 DAIGLE        712055  MALLARD       27 57 34.93000    .850
 1   69K4238 DAIGLE        69K4241 SOUTH DURHAM   47 32 33.13000    .850
 1   69K4241 SOUTH DURHAM  69K4238 DAIGLE          0  0  0.00000    .850
 1   69K4241 SOUTH DURHAM  712055  MALLARD        129 45  4.61000    .851
 1   712056  WICKHAM       69K4348 LEMAIRE         0  0  0.00000    .852
 1   712056  WICKHAM       69K4349 BREBOEUF       35 21  4.54000    .852
 1   712056  WICKHAM       712055  MALLARD        76 28 25.74000    .851
 1   712056  WICKHAM       69K4238 DAIGLE        157 51 13.81000    .850
 1   712055  MALLARD       69K4241 SOUTH DURHAM    0  0  0.00000    .851
 1   712055  MALLARD       69K4238 DAIGLE         30 39 56.09000    .850
 1   712055  MALLARD       712056  WICKHAM       101 19 33.90000    .851
 1   712055  MALLARD       69K4348 LEMAIRE       136 45 37.08000    .851
 1   712055  MALLARD       69K4349 BREBOEUF      146 47 25.39000    .852
 1   712055  MALLARD       69K4346 CHARLES       171 21  7.70000    .851
 1   69K4350 HEMMING       69K4346 CHARLES         0  0  0.00000    .917
 1   69K4350 HEMMING       712057  DRUMMOND      337 49 51.27000    .854
 1   712057  DRUMMOND      69K4346 CHARLES         0  0  0.00000    .856
 1   712057  DRUMMOND      69K4350 HEMMING         6  8 43.91000    .854
 1   712057  DRUMMOND      69K4349 BREBOEUF       42 17  1.97000    .853
 1   712057  DRUMMOND      69K4348 LEMAIRE        96 35 36.40000    .854
 1   69K4348 LEMAIRE       712057  DRUMMOND        0  0  0.00000    .854
 1   69K4348 LEMAIRE       69K4346 CHARLES        38 19 48.12000    .852
 1   69K4348 LEMAIRE       69K4349 BREBOEUF       68 33 46.39000    .855
 1   69K4348 LEMAIRE       712055  MALLARD        84 36 03.20000    .851
 1   69K4348 LEMAIRE       712056  WICKHAM       152 41 34.08000    .852
 1   69K4349 BREBOEUF      712055  MALLARD         0  0  0.00000    .852
 1   69K4349 BREBOEUF      69K4348 LEMAIRE       153 55 56.77000    .855
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 1   69K4349 BREBOEUF      712057  DRUMMOND      211  3 36.34000    .853
 1   69K4349 BREBOEUF      69K4346 CHARLES       263  4 13.69000    .858
 1   69K4346 CHARLES       69K4349 BREBOEUF        0  0  0.00000    .858
 1   69K4346 CHARLES       69K4348 LEMAIRE        40 37 44.90000    .852
 1   69K4346 CHARLES       712057  DRUMMOND       85 42 19.22000    .856
 1   69K4346 CHARLES       69K4350 HEMMING       294  1 10.24000    .917
 1   69K4346 CHARLES       712055  MALLARD       301 29 29.62000    .851
* 51F17   .70 0.0 0.01971  .20 .20FORGUESRL GEOD
 1   712057  DRUMMOND      712055  MALLARD         0  0  0.00000    .701
 1   712057  DRUMMOND      712056  WICKHAM        56 10 20.86000    .701
 1   712055  MALLARD       712056  WICKHAM         0  0  0.00000    .701
 1   712055  MALLARD       712057  DRUMMOND       58 25 52.51000    .701
 1   712056  WICKHAM       712055  MALLARD         0  0  0.00000    .701
 1   712056  WICKHAM       712057  DRUMMOND      294 36  9.46000    .701
* 51F18   .70 0.0 0.06971  .20 .20G-252    QLF
 1   09201   YAMASKA       69K4238 DAIGLE          0  0  0.00000    .700
 1   09201   YAMASKA       09206   ORFORD         26 14 32.07000    .700
 1   69K4238 DAIGLE        69K4241 SOUTH DURHAM    0  0  0.00000    .700
 1   69K4238 DAIGLE        69K4239 DUSSAULT       58 20  6.87000    .700
 1   69K4238 DAIGLE        09206   ORFORD         94 12 10.59000    .700
 1   69K4238 DAIGLE        09201   YAMASKA       224  4 44.22000    .700
 1   09206   ORFORD        09201   YAMASKA         0  0  0.00000    .700
 1   09206   ORFORD        69K4238 DAIGLE         23 52 57.48000    .700
 1   09206   ORFORD        69K4239 DUSSAULT       75 40 44.35000    .701
 1   09206   ORFORD        68K2071 SHERBROOKE    154  4 12.77000    .700
 1   09206   ORFORD        70K4631 HATLEY        195 23 38.11000    .700
 1   09206   ORFORD        70K4244 MAGOG         204 58 14.12000    .701
 1   09206   ORFORD        70K4245 AUSTIN        254 26  4.14000    .702
 1   09206   ORFORD        08207   OWLS HEAD     262  0 29.83000    .700
 1   69K4241 SOUTH DURHAM  69K4240 GALLUP HILL     0  0  0.00000    .702
 1   69K4241 SOUTH DURHAM  69K4239 DUSSAULT       57 39 21.24000    .700
 1   69K4241 SOUTH DURHAM  69K4238 DAIGLE        120 46 53.20000    .700
 1   69K4239 DUSSAULT      68K2071 SHERBROOKE      0  0  0.00000    .700
 1   69K4239 DUSSAULT      09206   ORFORD         63 20 52.66000    .701
 1   69K4239 DUSSAULT      69K4238 DAIGLE        155 41  2.20000    .700
 1   69K4239 DUSSAULT      69K4241 SOUTH DURHAM  214 13 25.92000    .700
 1   69K4239 DUSSAULT      69K4240 GALLUP HILL   248 10 34.54000    .701
 1   69K4239 DUSSAULT      69K4243 LAROCHELLE    301 48 33.39000    .701
 1   69K4239 DUSSAULT      68K2073 BEAUVOIR      333 40  3.05000    .700
 1   69K4240 GALLUP HILL   69K4242 PINNACLE        0  0  0.00000    .701
 1   69K4240 GALLUP HILL   69K4243 LAROCHELLE     83 54 31.06000    .701
 1   69K4240 GALLUP HILL   69K4239 DUSSAULT      131 17 30.49000    .701
 1   69K4240 GALLUP HILL   69K4241 SOUTH DURHAM  219 41  1.14000    .702
 1   69K4242 PINNACLE      70K4634 ASBESTOS        0  0  0.00000    .703
 1   69K4242 PINNACLE      70K4633 CHAPMAN        57 32 43.37000    .700
 1   69K4242 PINNACLE      69K4243 LAROCHELLE    127 18 20.41000    .700
 1   69K4242 PINNACLE      69K4240 GALLUP HILL   171 10 14.78000    .701
 1   69K4243 LAROCHELLE    69K4242 PINNACLE        0  0  0.00000    .700
 1   69K4243 LAROCHELLE    70K4633 CHAPMAN        68 32 34.91000    .700
 1   69K4243 LAROCHELLE    68K2073 BEAUVOIR      108 22 53.91000    .701
 1   69K4243 LAROCHELLE    68K2071 SHERBROOKE    137 48 47.34000    .700
 1   69K4243 LAROCHELLE    69K4239 DUSSAULT      228 47 21.19000    .701
 1   69K4243 LAROCHELLE    69K4240 GALLUP HILL   307 46 24.87000    .701
 1   68K2073 BEAUVOIR      68K2071 SHERBROOKE      0  0  0.00000    .702
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1	68K2073 BEAUVOIR	69K4239 DUSSAULT	83 5 20.51000	.700
1	68K2073 BEAUVOIR	69K4243 LAROCHELLE	110 49 24.26000	.701
1	68K2073 BEAUVOIR	70K4632 MARTIN	299 2 22.45000	.700
1	68K2071 SHERBROOKE	69K4243 LAROCHELLE	0 0 0.00000	.700
1	68K2071 SHERBROOKE	68K2073 BEAUVOIR	39 44 44.10000	.702
1	68K2071 SHERBROOKE	70K4633 CHAPMAN	67 7 11.58000	.700
1	68K2071 SHERBROOKE	70K4632 MARTIN	130 36 5.77000	.700
1	68K2071 SHERBROOKE	70K4631 HATLEY	201 42 34.67000	.701
1	68K2071 SHERBROOKE	08207 OWLS HEAD	252 29 39.97000	.700
1	68K2071 SHERBROOKE	70K4244 MAGOG	259 31 22.71000	.701
1	68K2071 SHERBROOKE	09206 ORFORD	290 54 20.21000	.700
1	68K2071 SHERBROOKE	69K4239 DUSSAULT	329 9 59.13000	.700
1	70K4244 MAGOG	08207 OWLS HEAD	0 0 0.00000	.700
1	70K4244 MAGOG	70K4245 AUSTIN	34 30 27.98000	.702
1	70K4244 MAGOG	09206 ORFORD	95 8 2.19000	.701
1	70K4244 MAGOG	68K2071 SHERBROOKE	192 51 3.39000	.701
1	70K4245 AUSTIN	08207 OWLS HEAD	0 0 0.00000	.701
1	70K4245 AUSTIN	09206 ORFORD	166 40 21.85000	.702
1	70K4245 AUSTIN	70K4244 MAGOG	236 34 57.11000	.702
1	70K4633 CHAPMAN	68K2071 SHERBROOKE	0 0 0.00000	.700
1	70K4633 CHAPMAN	69K4243 LAROCHELLE	43 36 38.24000	.700
1	70K4633 CHAPMAN	69K4242 PINNACLE	85 18 27.31000	.700
1	70K4633 CHAPMAN	70K4634 ASBESTOS	100 36 48.69000	.700
1	70K4633 CHAPMAN	09205 HAM	150 27 22.68000	.700
1	70K4633 CHAPMAN	70K4635 WEEDON	208 44 22.87000	.701
1	70K4633 CHAPMAN	70K4632 MARTIN	316 37 6.26000	.700
1	70K4632 MARTIN	70K4635 WEEDON	0 0 0.00000	.700
1	70K4632 MARTIN	09208 MEGANTIC	46 29 13.96000	.700
1	70K4632 MARTIN	09207 HEREFORD	150 15 5.41000	.700
1	70K4632 MARTIN	70K4631 HATLEY	206 11 12.76000	.700
1	70K4632 MARTIN	68K2071 SHERBROOKE	258 55 12.17000	.700
1	70K4632 MARTIN	70K4633 CHAPMAN	332 3 25.58000	.700
1	70K4631 HATLEY	68K2071 SHERBROOKE	0 0 0.00000	.701
1	70K4631 HATLEY	70K4632 MARTIN	56 9 32.26000	.700
1	70K4631 HATLEY	09207 HEREFORD	116 36 44.09000	.700
1	70K4631 HATLEY	08207 OWLS HEAD	260 44 22.26000	.700
1	70K4631 HATLEY	09206 ORFORD	310 31 6.17000	.700
1	70K4634 ASBESTOS	09205 HAM	0 0 0.00000	.701
1	70K4634 ASBESTOS	70K4633 CHAPMAN	58 54 59.51000	.700
1	70K4634 ASBESTOS	69K4242 PINNACLE	166 3 56.41000	.703
1	09205 HAM	70K4639 MOISAN	0 0 0.00000	.701
1	09205 HAM	70K4638 COULOMBE	38 10 20.70000	.701
1	09205 HAM	09208 MEGANTIC	113 16 24.69000	.700
1	09205 HAM	70K4635 WEEDON	118 8 13.44000	.700
1	09205 HAM	70K4633 CHAPMAN	168 3 43.54000	.700
1	09205 HAM	70K4634 ASBESTOS	239 18 10.09000	.701
1	70K4635 WEEDON	09205 HAM	0 0 0.00000	.700
1	70K4635 WEEDON	70K4638 COULOMBE	33 19 38.48000	.700
1	70K4635 WEEDON	71K6154 STORNOWAY	108 55 30.15000	.701
1	70K4635 WEEDON	09208 MEGANTIC	171 54 56.82000	.700
1	70K4635 WEEDON	70K4632 MARTIN	244 1 45.40000	.700
1	70K4635 WEEDON	70K4633 CHAPMAN	288 12 29.60000	.701
1	70K4639 MOISAN	70K4638 COULOMBE	0 0 0.00000	.703
1	70K4639 MOISAN	09205 HAM	73 39 27.26000	.701

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1  70K4638 COULOMBE     71K6159 LAPOINTE        0  0  0.00000    .703
1  70K4638 COULOMBE     66KP115 CARIBOU E-15   71 29 28.41000    .701
1  70K4638 COULOMBE     71K6155 STE PRAXEDE   126 24 50.10000    .701
1  70K4638 COULOMBE     71K6154 STORNOWAY     175  1 59.09000    .700
1  70K4638 COULOMBE     70K4635 WEEDON        221 38 54.81000    .700
1  70K4638 COULOMBE     09205   HAM           288 21 25.12000    .701
1  70K4638 COULOMBE     70K4639 MOISAN        356 31 35.59000    .703
1  71K6159 LAPOINTE     71K6165 VIANNEY         0  0  0.00000    .701
1  71K6159 LAPOINTE     66KP115 CARIBOU E-15   60 35 18.24000    .701
1  71K6159 LAPOINTE     70K4638 COULOMBE      141 55 10.70000    .703
1  71K6154 STORNOWAY    71K6155 STE PRAXEDE     0  0  0.00000    .701
1  71K6154 STORNOWAY    71K6156 SEBASTIEN      80 31 13.62000    .700
1  71K6154 STORNOWAY    70K4637 GILBERT       128 12 39.02000    .701
1  71K6154 STORNOWAY    09208   MEGANTIC      171 40 32.86000    .700
1  71K6154 STORNOWAY    70K4635 WEEDON        251 44 19.65000    .701
1  71K6154 STORNOWAY    70K4638 COULOMBE      309 31 32.90000    .700
1  71K6155 STE PRAXEDE  692012  ADSTOCK         0  0  0.00000    .701
1  71K6155 STE PRAXEDE  692010  GRELOTS        52 48  9.24000    .701
1  71K6155 STE PRAXEDE  71K6156 SEBASTIEN     117 37  8.74000    .700
1  71K6155 STE PRAXEDE  71K6154 STORNOWAY     170 23 33.07000    .701
1  71K6155 STE PRAXEDE  70K4638 COULOMBE      251 17 57.87000    .701
1  71K6155 STE PRAXEDE  66KP115 CARIBOU E-15  305  5 56.85000    .701
1  692010  GRELOTS      692009  HONORE          0  0  0.00000    .701
1  692010  GRELOTS      71K6156 SEBASTIEN      57  3 46.11000    .700
1  692010  GRELOTS      71K6155 STE PRAXEDE   132 53 33.64000    .701
1  692010  GRELOTS      692012  ADSTOCK       184 23 49.21000    .701
* 51F20    .70  0.0  0.01971  .20  .20FORGUESRL GEOD
1  712050  BON CONSEIL  712055  MALLARD         0  0  0.00000    .700
1  712050  BON CONSEIL  712056  WICKHAM        33  3 44.15000    .700
1  712050  BON CONSEIL  712057  DRUMMOND       50 10 11.76000    .701
1  712050  BON CONSEIL  712051  ST MAJORIQUE   66 59 55.20000    .701
1  712051  ST MAJORIQUE 712050  BON CONSEIL     0  0  0.00000    .701
1  712051  ST MAJORIQUE 712057  DRUMMOND       28 21  2.07000    .703
1  712051  ST MAJORIQUE 712055  MALLARD        60 32 35.67000    .700
1  712051  ST MAJORIQUE 712056  WICKHAM       101 58  5.17000    .701
1  712050  BON CONSEIL  712057  DRUMMOND        0  0  0.00000    .701
1  712050  BON CONSEIL  712051  ST MAJORIQUE   16 49 43.04000    .701
1  712050  BON CONSEIL  712055  MALLARD       309 49 48.11000    .700
1  712050  BON CONSEIL  712056  WICKHAM       342 53 33.11000    .700
1  712057  DRUMMOND     712050  BON CONSEIL     0  0  0.00000    .701
1  712057  DRUMMOND     712055  MALLARD        92 18 10.08000    .701
1  712057  DRUMMOND     712056  WICKHAM       148 28 26.40000    .701
1  712051  ST MAJORIQUE 712053  ST ZEPHIRIN     0  0  0.00000    .701
1  712051  ST MAJORIQUE 712050  BON CONSEIL    74 40 32.39000    .701
1  712057  DRUMMOND     712055  MALLARD         0  0  0.00000    .701
1  712057  DRUMMOND     712050  BON CONSEIL   267 41 50.59000    .701
1  712055  MALLARD      712056  WICKHAM         0  0  0.00000    .701
1  712055  MALLARD      712050  BON CONSEIL    95 57 33.41000    .700
1  712057  DRUMMOND     712056  WICKHAM         0  0  0.00000    .701
1  712057  DRUMMOND     712051  ST MAJORIQUE   76 42 19.21000    .703
1  712057  DRUMMOND     712050  BON CONSEIL   211 31 33.55000    .701
1  712056  WICKHAM      712051  ST MAJORIQUE    0  0  0.00000    .701
1  712056  WICKHAM      712050  BON CONSEIL    44  5 44.55000    .700
1  712056  WICKHAM      712055  MALLARD        95  4 29.34000    .701
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 1   712055  MALLARD      712051  ST MAJORIQUE     0  0  0.00000    .700
 1   712055  mallard      712050  bon conseil     52 27 30.03000    .700
 1   712056  WICKHAM      712055  MALLARD          0  0  0.00000    .701
 1   712056  WICKHAM      712050  BON CONSEIL    309  1 15.17000    .700
 1   712055  MALLARD      712057  DRUMMOND         0  0  0.00000    .701
 1   712055  MALLARD      712050  BON CONSEIL     37 31 41.70000    .700
 1   712055  MALLARD      712056  WICKHAM        301 34  9.60000    .701
 1   712056  WICKHAM      712057  DRUMMOND         0  0  0.00000    .701
 1   712056  WICKHAM      712050  BON CONSEIL     14 25  4.78000    .700
 1   712056  WICKHAM      712055  MALLARD         65 23 48.98000    .701
* 51F19    .70  0.0  0.01969  .20  .20SELLEYAD  GEOD
 1   692010  GRELOTS      692012  ADSTOCK          0  0  0.00000    .701
 1   692010  GRELOTS      692011  BROUGHTON       51 35 38.80000    .701
 1   692010  GRELOTS      692009  HONORE         175 36 10.89000    .701
* 51F21    .70  0.0  0.01965  .20  .20G-22     QLF
 1   652402  BROMONT      08200   ST ARMAND        0  0  0.00000    .700
 1   652402  BROMONT      652401  FARNHAM         75 14 59.77000    .700
 1   652402  BROMONT      09201   YAMASKA        117 28 38.98000    .700
 1   652402  BROMONT      09206   ORFORD         248 43  2.52000    .700
 1   09206   ORFORD       652402  BROMONT          0  0  0.00000    .700
 1   09206   ORFORD       09201   YAMASKA         21 35 53.30000    .700
 1   09206   ORFORD       08200   ST ARMAND      327 38 30.11000    .700
 1   652401  FARNHAM      09201   YAMASKA          0  0  0.00000    .701
 1   652401  FARNHAM      652402  BROMONT         69 29 59.23000    .700
 1   652401  FARNHAM      08200   ST ARMAND      126 21 55.30000    .700
 1   09201   YAMASKA      09206   ORFORD           0  0  0.00000    .700
 1   09201   YAMASKA      652402  BROMONT         27  9 43.36000    .700
 1   09201   YAMASKA      08200   ST ARMAND       60 23  6.94000    .700
 1   09201   YAMASKA      652401  FARNHAM         95 26  7.78000    .701
* 51F22    .60  0.0  0.00917  .20  .20BIGGERCA  GEOD
 1   09206   ORFORD       08200   ST ARMAND        0  0  0.00000    .600
 1   09206   ORFORD       09201   YAMASKA         53 57 23.47000    .600
 1   09206   ORFORD       09205   HAM            168 22 58.78000    .600
 1   09206   ORFORD       09208   MEGANTIC       206 44 27.02000    .600
 1   09206   ORFORD       09207   HEREFORD       243 30 22.34000    .600
 1   09206   ORFORD       08207   OWLS HEAD      315 57 53.29000    .600
 1   09201   YAMASKA      08200   ST ARMAND        0  0  0.00000    .600
 1   09201   YAMASKA      09202   DUSABLE        176 15 45.12000    .600
 1   09201   YAMASKA      09204   CARMEL         201 58 27.05000    .600
 1   09201   YAMASKA      09205   HAM            260 49 45.47000    .600
 1   09201   YAMASKA      09206   ORFORD         299 36 53.83000    .600
 1   09205   HAM          09209   THETFORD         0  0  0.00000    .600
 1   09205   HAM          09210   LINIERE         56 58  9.61000    .600
 1   09205   HAM          14200   STRATFORD       59  3 26.46000    .600
 1   09205   HAM          09208   MEGANTIC       103 21 26.36000    .600
 1   09205   HAM          09207   HEREFORD       148  1 39.92000    .600
 1   09205   HAM          09206   ORFORD         191 49 56.63000    .600
 1   09205   HAM          09201   YAMASKA        218 37 20.09000    .600
 1   09205   HAM          09202   DUSABLE        261 27 29.72000    .600
 1   09205   HAM          09204   CARMEL         285 58  2.16000    .600
* 51F12    .70  0.0  0.01914  .20  .20BIGGERCA  GEOD
 1   14200   STRATFORD    09205   HAM              0  0  0.00000    .700
 1   14200   STRATFORD    09209   THETFORD        81 29 40.13000    .700
* 51F04   1.20  0.0  0.01909  .20  .20BIGGERCA  GEOD
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 1  09205  HAM        09209  THETFORD       0  0  0.00000    1.200
 1  09205  HAM        09216  ARTHABASKA   295 55 51.30000    1.200
* 51F23   .85  0.0  0.01974  .20  .20G-272   QLF
 1  71K6159 LAPOINTE    70K4638 COULOMBE      0  0  0.00000    .853
 1  71K6159 LAPOINTE    09205  HAM           71 35  7.76000    .851
 1  71K6159 LAPOINTE    72K7457 SEVIGNY     158 34 38.99000    .851
 1  71K6159 LAPOINTE    71K6165 VIANNEY     218  4 52.37000    .850
 1  72K7457 SEVIGNY     72K7455 VICTORIAVILLE  0  0  0.00000   .851
 1  72K7457 SEVIGNY     09216  ARTHABASKA    18  1 30.66000    .851
 1  72K7457 SEVIGNY     71K6165 VIANNEY      87 43 47.83000    .851
 1  72K7457 SEVIGNY     71K6159 LAPOINTE    171 17 16.76000    .851
 1  72K7457 SEVIGNY     09205  HAM          219 53  4.95000    .851
 1  72K7457 SEVIGNY     70K4634 ASBESTOS    278 55 28.28000    .850
 1  70K4638 COULOMBE    09205  HAM            0  0  0.00000    .851
 1  70K4638 COULOMBE    71K6159 LAPOINTE     71 38 33.63000    .853
 1  09205  HAM          70K4634 ASBESTOS      0  0  0.00000    .850
 1  09205  HAM          72K7457 SEVIGNY      77 41  6.70000    .851
 1  09205  HAM          71K6159 LAPOINTE    122  5 47.43000    .851
 1  09205  HAM          70K4638 COULOMBE    158 52  6.90000    .851
 1  70K4634 ASBESTOS    72K7457 SEVIGNY       0  0  0.00000    .850
 1  70K4634 ASBESTOS    09205  HAM           43 16 30.97000    .850
* 51F24   2.00  0.0  0.01974  .20  .20G-333-1  QLF
 1  72K7462 KINGSEY FALLS  72K7463 ST FELIX      0  0  0.00000   2.000
 1  72K7462 KINGSEY FALLS  09216  ARTHABASKA   182 54 18.41000   2.000
 1  72K7462 KINGSEY FALLS  70K4634 ASBESTOS    283 37  1.02000   2.000
 1  70K4634 ASBESTOS    69K4242 PINNACLE      0  0  0.00000   2.001
 1  70K4634 ASBESTOS    72K7462 KINGSEY FALLS  76 21 35.35000   2.000
 1  70K4634 ASBESTOS    09205  HAM          193 56  4.07000   2.000
 1  09205  HAM          69K4242 PINNACLE      0  0  0.00000   2.000
 1  09205  HAM          72K7463 ST FELIX     15 50 11.28000   2.000
 1  09205  HAM          09216  ARTHABASKA    70 25 24.97000   2.000
 1  72K7463 ST FELIX    72K7462 KINGSEY FALLS  0  0  0.00000   2.000
 1  72K7463 ST FELIX    69K4240 GALLUP HILL  144  1 10.17000   2.000
 1  72K7463 ST FELIX    712055  MALLARD     222 30 17.05000   2.000
 1  69K4240 GALLUP HILL  72K7463 ST FELIX     0  0  0.00000   2.000
 1  69K4240 GALLUP HILL  69K4242 PINNACLE    54 24 31.70000   2.000
 1  69K4240 GALLUP HILL  712055  MALLARD    312 40 43.99000   2.000
* 52T33   5.00  5.0  0.01969  .20  .20G-213   QLF
 2  69K4238 DAIGLE      69K4241 SOUTH DURHAM  4   22248.805    12.199
 2  69K4238 DAIGLE      712055  MALLARD      4   33538.096    17.500
 2  69K4241 SOUTH DURHAM  712055  MALLARD    4   14620.739     8.861
 2  69K4348 LEMAIRE     712056  WICKHAM      4    9938.004     7.055
 2  712056  WICKHAM     69K4349 BREBOEUF     4   11356.349     7.571
 2  712055  MALLARD     69K4348 LEMAIRE      4   16666.069     9.722
 2  712055  MALLARD     69K4349 BREBOEUF     4   10478.186     7.247
 2  69K4238 DAIGLE      712056  WICKHAM      4   32007.383    16.768
 2  712056  WICKHAM     712055  MALLARD      4   15903.665     9.397
* 52G66   1.00  1.2  0.01969  .20  .20G-213   QLF
 2  712055  MALLARD     69K4346 CHARLES      4   12198.025     1.795
 2  69K4348 LEMAIRE     712057  DRUMMOND     4    6830.520     1.324
 2  69K4348 LEMAIRE     69K4346 CHARLES      4    9583.193     1.550
 2  69K4348 LEMAIRE     69K4349 BREBOEUF     4    6605.216     1.307
 2  69K4349 BREBOEUF    69K4346 CHARLES      4    5107.571     1.207
 2  69K4349 BREBOEUF    712057  DRUMMOND     4    7570.098     1.380
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 2  69K4346 CHARLES      69K4350 HEMMING        4   1697.535    1.059
 2  69K4346 CHARLES      712057  DRUMMOND       4   5982.974    1.263
 2  69K4350 HEMMING      712057  DRUMMOND       4   7520.554    1.376
* 52T34   3.00  3.0  0.01971  .20  .20FORGUESRL GEOD
 2  712056  WICKHAM      712057  DRUMMOND       1  16311.219    5.747
 2  712056  WICKHAM      712055  MALLARD        1  15903.735    5.643
 2  712055  MALLARD      712057  DRUMMOND       1  17406.470    6.029
* 52T03   5.00  5.0  0.06971  .20  .20G-252      QLF
 2  68K2073 BEAUVOIR     70K4632 MARTIN         4  25958.954   13.911
 2  70K4632 MARTIN       09207   HEREFORD       4  25114.522   13.516
 2  70K4632 MARTIN       70K4631 HATLEY         4  25855.606   13.863
 2  70K4631 HATLEY       09207   HEREFORD       4  23915.600   12.961
 2  09201   YAMASKA      09206   ORFORD         4  51349.497   26.155
 2  09201   YAMASKA      69K4238 DAIGLE         4  27087.768   14.439
 2  69K4238 DAIGLE       69K4241 SOUTH DURHAM   4  22248.919   12.199
 2  69K4238 DAIGLE       69K4239 DUSSAULT       4  23266.547   12.665
 2  69K4238 DAIGLE       09206   ORFORD         4  29589.185   15.615
 2  09206   ORFORD       69K4239 DUSSAULT       4  17353.488   10.015
 2  09206   ORFORD       68K2071 SHERBROOKE     4  25042.285   13.483
 2  09206   ORFORD       70K4631 HATLEY         4  32937.049   17.210
 2  09206   ORFORD       70K4244 MAGOG          4  13167.713    8.268
 2  09206   ORFORD       70K4245 AUSTIN         4  12221.058    7.896
 2  09206   ORFORD       08207   OWLS HEAD      4  28074.228   14.902
 2  69K4241 SOUTH DURHAM 69K4240 GALLUP HILL    4  12404.558    7.971
 2  69K4241 SOUTH DURHAM 69K4239 DUSSAULT       4  22201.299   12.177
 2  69K4239 DUSSAULT     68K2071 SHERBROOKE     4  27441.676   14.605
 2  69K4239 DUSSAULT     69K4240 GALLUP HILL    4  18763.557   10.634
 2  69K4239 DUSSAULT     69K4243 LAROCHELLE     4  14067.426    8.634
 2  69K4239 DUSSAULT     68K2073 BEAUVOIR       4  26069.828   13.963
 2  69K4240 GALLUP HILL  69K4242 PINNACLE       4  17557.730   10.106
 2  69K4240 GALLUP HILL  69K4243 LAROCHELLE     4  15392.461    9.182
 2  69K4242 PINNACLE     70K4634 ASBESTOS       4   8537.313    6.580
 2  69K4242 PINNACLE     70K4633 CHAPMAN        4  30904.295   16.242
 2  69K4242 PINNACLE     69K4243 LAROCHELLE     4  22087.271   12.125
 2  69K4243 LAROCHELLE   70K4633 CHAPMAN        4  31156.248   16.361
 2  69K4243 LAROCHELLE   68K2073 BEAUVOIR       4  15954.418    9.419
 2  69K4243 LAROCHELLE   68K2071 SHERBROOKE     4  23323.516   12.691
 2  68K2073 BEAUVOIR     68K2071 SHERBROOKE     4  12262.201    7.916
 2  68K2071 SHERBROOKE   70K4633 CHAPMAN        4  31626.690   16.586
 2  68K2071 SHERBROOKE   70K4632 MARTIN         4  22698.097   12.404
 2  68K2071 SHERBROOKE   70K4631 HATLEY         4  21747.938   11.971
 2  68K2071 SHERBROOKE   08207   OWLS HEAD      4  42989.262   22.068
 2  68K2071 SHERBROOKE   70K4244 MAGOG          4  19608.686   11.009
 2  70K4244 MAGOG        08207   OWLS HEAD      4  23652.302   12.840
 2  70K4244 MAGOG        70K4245 AUSTIN         4  10647.587    7.309
 2  70K4245 AUSTIN       08207   OWLS HEAD      4  16056.795    9.459
 2  70K4633 CHAPMAN      70K4634 ASBESTOS       4  27291.916   14.534
 2  70K4633 CHAPMAN      09205   HAM            4  24683.525   13.318
 2  70K4633 CHAPMAN      70K4632 MARTIN         4  29571.329   15.609
 2  70K4632 MARTIN       70K4635 WEEDON         4  40381.821   20.802
 2  70K4634 ASBESTOS     09205   HAM            4  22030.464   12.098
 2  09205   HAM          70K4639 MOISAN         4  12790.996    8.122
 2  09205   HAM          70K4638 COULOMBE       4  13223.970    8.293
 2  09205   HAM          70K4635 WEEDON         4  22105.434   12.133
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2   09205  HAM           09209  THETFORD      4   45751.072    23.415
2   70K4635 WEEDON       70K4638 COULOMBE     4   23696.215    12.862
2   70K4635 WEEDON       71K6154 STORNOWAY    4   20354.838    11.342
2   70K4639 MOISAN       70K4638 COULOMBE     4   8516.299     6.573
2   70K4638 COULOMBE     71K6159 LAPOINTE     4   8343.637     6.517
2   70K4638 COULOMBE     66KP115 CARIBOU E-15  4  18055.555    10.323
2   70K4638 COULOMBE     71K6155 STE PRAXEDE  4   21190.456    11.718
2   70K4638 COULOMBE     71K6154 STORNOWAY    4   27127.252    14.458
2   71K6159 LAPOINTE     71K6165 VIANNEY      4   20404.081    11.364
2   71K6159 LAPOINTE     66KP115 CARIBOU E-15  4  17319.615    10.003
2   71K6154 STORNOWAY    71K6155 STE PRAXEDE  4   20613.706    11.458
2   71K6154 STORNOWAY    71K6156 SEBASTIEN    4   22553.585    12.337
2   71K6154 STORNOWAY    70K4637 GILBERT      4   21159.368    11.704
2   71K6155 STE PRAXEDE  692012  ADSTOCK      4   14759.889    8.917
2   71K6155 STE PRAXEDE  692010  GRELOTS      4   18269.250    10.417
2   71K6155 STE PRAXEDE  71K6156 SEBASTIEN    4   27938.314    14.837
2   71K6155 STE PRAXEDE  66KP115 CARIBOU E-15  4  18311.539    10.435
* 52T02   6.12  6.1  0.06971  .20  .20G-252    QLF
2   692010  GRELOTS      692009  HONORE       4   14873.258    10.947
* 52G83   1.40  1.7  0.06971  .20  .20G-252    QLF
2   70K4635 WEEDON       70K4633 CHAPMAN      4   19884.200    3.669
* 52T43   1.50  3.0  0.01969  .20  .20SELLEYAD  GEOD
2   692010  GRELOTS      692011  BROUGHTON    1   18091.670    5.637
2   692010  GRELOTS      692012  ADSTOCK      1   15021.754    4.757
* 52T35   3.00  3.0  0.01971  .20  .20FORGUESRL GEOD
2   712050  BON CONSEIL  712053  ST ZEPHIRIN  1   23639.304    7.705
2   712050  BON CONSEIL  712051  ST MAJORIQUE 1   20623.942    6.882
2   712050  BON CONSEIL  712057  DRUMMOND     1   13807.056    5.122
2   712050  BON CONSEIL  712056  WICKHAM      2   28993.797    9.205
2   712051  ST MAJORIQUE 712057  DRUMMOND     1   8417.650     3.932
2   712051  ST MAJORIQUE 712053  ST ZEPHIRIN  1   18224.873    6.243
2   712056  WICKHAM      712051  ST MAJORIQUE 1   16545.896    5.807
2   712055  MALLARD      712051  ST MAJORIQUE 1   23942.647    7.789
* 52T36   6.00  3.0  0.01971  .20  .20FORGUESRL GEOD
2   712055  MALLARD      712050  BON CONSEIL  1   22647.652    9.069
* 52T32   3.70  3.7  0.01965  .20  .20G-22    QLF
2   09201   YAMASKA      08200   ST ARMAND    4   45568.921    17.262
2   09201   YAMASKA      652401  FARNHAM      4   18037.649    7.635
2   09201   YAMASKA      652402  BROMONT      4   25135.574    10.012
2   652402  BROMONT      09206   ORFORD       4   31174.455    12.115
2   652402  BROMONT      652401  FARNHAM      4   24932.164    9.941
* 52T01   5.00  5.0  0.01972  .20  .20G-272    QLF
2   71K6159 LAPOINTE     09205   HAM          4   13226.475    8.295
2   72K7457 SEVIGNY      72K7455 VICTORIAVILLE 4  18125.702    10.353
2   72K7457 SEVIGNY      09216   ARTHABASKA   4   18296.441    10.428
2   72K7457 SEVIGNY      09205   HAM          4   17609.832    10.128
2   72K7457 SEVIGNY      70K4634 ASBESTOS     4   25096.466    13.510
2   71K6159 LAPOINTE     72K7457 SEVIGNY      4   12340.221    7.946
* 52G84   1.40  1.7  0.01974  .20  .20G-333-1  QLF
2   72K7463 ST FELIX     69K4240 GALLUP HILL  4   18293.961    3.422
2   72K7463 ST FELIX     712055  MALLARD      4   16581.838    3.160
* 52T04   5.00  5.0  0.01974  .20  .20G-333-1  QLF
2   72K7462 KINGSEY FALLS  72K7463 ST FELIX   4   14338.006    8.745
2   72K7462 KINGSEY FALLS  70K4634 ASBESTOS   4   20793.098    11.539
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 2   72K7463 ST FELIX      69K4242 PINNACLE     4   16404.403      9.609
 2   72K7463 ST FELIX      69K4240 GALLUP HILL  4   18293.913     10.428
* 52T05   5.20  5.2  0.0      .20  .20G-257     SGQ
 2   09205  HAM          14200  STRATFORD     4   29394.835     16.146
* 53???   .80  .2  1.0      .20  .20          GEOD
 3   09206  ORFORD        09201  YAMASKA       287  4 48.92700    1.089
* 51F25   .70  0.0  0.06971   .20  .20G-252    QLF
 1   09208  MEGANTIC      09207  HEREFORD       0  0  0.00000    .700
 1   09208  MEGANTIC      70K4632 MARTIN        26  7  1.87000    .700
 1   09208  MEGANTIC      70K4635 WEEDON        87 31  3.38000    .700
 1   09208  MEGANTIC      09205  HAM           90 44 18.88000    .700
 1   09208  MEGANTIC      71K6154 STORNOWAY    124 27 51.03000    .700
 1   09208  MEGANTIC      70K4637 GILBERT      168 57 40.27000    .701
 1   09208  MEGANTIC      65K0335 CROIX        193 15 38.35000    .700
 1   08207  OWLS HEAD     09206  ORFORD         0  0  0.00000    .700
 1   08207  OWLS HEAD     70K4245 AUSTIN         5 45 13.57000    .701
 1   08207  OWLS HEAD     70K4244 MAGOG         27 49 43.37000    .700
 1   08207  OWLS HEAD     68K2071 SHERBROOKE    33 39  5.61000    .700
 1   08207  OWLS HEAD     70K4631 HATLEY        63 36 25.33000    .700
 1   08207  OWLS HEAD     09207  HEREFORD       78 22 59.22000    .700
 1   09207  HEREFORD      08207  OWLS HEAD      0  0  0.00000    .700
 1   09207  HEREFORD      70K4631 HATLEY        21  5 48.54000    .700
 1   09207  HEREFORD      70K4632 MARTIN        84 42 29.48000    .700
 1   09207  HEREFORD      09208  MEGANTIC      134 49 36.75000    .700
* 51F26   .70  0.0  0.01965  .20  .20G-22     QLF
 1   08200  ST ARMAND     09201  YAMASKA        0  0  0.00000    .700
 1   08200  ST ARMAND     652402  BROMONT       29 18  .75000    .700
 1   08200  ST ARMAND     09206  ORFORD         65 39 36.76000    .700
 1   08200  ST ARMAND     652401  FARNHAM      341 24 53.93000    .700
* 51F27   .60  0.0  0.00917  .20  .20BIGGERCA  GEOD
 1   08200  ST ARMAND     09201  YAMASKA        0  0  0.00000    .600
 1   08200  ST ARMAND     09206  ORFORD         65 39 37.18000    .600
 1   08200  ST ARMAND     08207  OWLS HEAD      99 46  8.20000    .600
 1   08207  OWLS HEAD     08200  ST ARMAND      0  0  0.00000    .600
 1   08207  OWLS HEAD     09206  ORFORD        101 51 26.66000    .600
 1   08207  OWLS HEAD     09207  HEREFORD      180 14 25.69000    .600
 1   09207  HEREFORD      08207  OWLS HEAD      0  0  0.00000    .600
 1   09207  HEREFORD      09206  ORFORD         29  9 31.30000    .600
 1   09207  HEREFORD      09205  HAM           90 13 59.17000    .600
 1   09207  HEREFORD      09208  MEGANTIC      134 49 35.51000    .600
 1   09208  MEGANTIC      09205  HAM            0  0  0.00000    .600
 1   09208  MEGANTIC      09209  THETFORD       34  2 22.24000    .600
 1   09208  MEGANTIC      09210  LINIERE        99 52 16.74000    .600
 1   09208  MEGANTIC      09207  HEREFORD      269 15 43.17000    .600
 1   09208  MEGANTIC      09206  ORFORD        306 49 50.20000    .600
* 52T00   5.00  5.0  0.06971  .20  .20G-252    QLF
 2   08207  OWLS HEAD     70K4631 HATLEY      4   33747.646     17.599
 2   08207  OWLS HEAD     09207  HEREFORD     4   54940.557     27.920
 2   09208  MEGANTIC      65K0335 CROIX       4   23201.228     12.631
 2   09208  MEGANTIC      70K4637 GILBERT     4   20774.777     11.528
 2   09208  MEGANTIC      71K6154 STORNOWAY   4   30177.294     15.894
 2   09208  MEGANTIC      09205  HAM          4   55331.838     28.111
 2   09208  MEGANTIC      70K4635 WEEDON      4   33363.916     17.413
 2   09208  MEGANTIC      70K4632 MARTIN      4   43778.057     22.450
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* 52T06   6.12 6.1 0.06971  .20 .20G-252   QLF
 2  09208 MEGANTIC     09207 HEREFORD    4  55407.113    34.344
* 52T37   3.70 3.7 0.01965  .20 .20G-22    QLF
 2  08200 ST ARMAND    652401 FARNHAM     4  32503.501     12.583
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Table 7.6 NETAN listing of reliability analysis results for real 2D network.

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NETAN: Network Analysis (Version 21 Nov 90)
Network Strength Analysis
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Piece-Wise Linear Approximation -- Connected Stations

Input network data file : [ong.work.gsc]
Real 2D Network. Sigma record commented out. November 16, 1990.


Station     Name
         Lat (DMS), Long (DMS), Ht (m)
         Strength in Rotation: Lat/Lon, Lat/Ht, Lon/Ht (rad)
            Obs # and Type
         Strength in Shear: Lat/Lon, Lat/Ht, Lon/Ht (strain)
            Obs # and Type
         Strength in Scale: (strain)
            Obs # and Type

  1    09206       ORFORD
         45 18 43.080825   -72 14 30.207541      823.910000
         0.2291992303E-05  0.0000000000E+00   0.0000000000E+00
              216 dir      0            0
         0.6780170129E-06  0.0000000000E+00   0.0000000000E+00
              408 dis      0            0
         0.1048841776E-05
              408 dis

  2    08200       ST ARMAND
         45  2 46.871261   -72 44 20.953894      683.340000
         0.1787265735E-05  0.0000000000E+00   0.0000000000E+00
              406 dis      0            0
         0.1887607108E-05  0.0000000000E+00   0.0000000000E+00
              408 dis      0            0
         0.2712184001E-05
              408 dis

  3    09201       YAMASKA
         45 26 45.361290   -72 52  8.645262      392.290000
         0.2439381765E-05  0.0000000000E+00   0.0000000000E+00
              225 dir      0            0
         0.3127375182E-05  0.0000000000E+00   0.0000000000E+00
              224 dir      0            0
         0.3791708636E-05
              224 dir

  4    652401      FARNHAM
         45 17 43.897511   -72 57 19.611919      48.700000
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```
   0.5001618512E-05   0.0000000000E+00   0.0000000000E+00
       204 dir         0             0
   0.3307516240E-05   0.0000000000E+00   0.0000000000E+00
       202 dir         0             0
   0.3517456435E-05
       409 dis

5   652402      BROMONT
    45 17 21.072496   -72 38 16.135074       524.090000
  -0.1843621826E-05   0.0000000000E+00   0.0000000000E+00
       221 dir         0             0
   0.1889051795E-05   0.0000000000E+00   0.0000000000E+00
       293 dir         0             0
   0.2437763748E-05
       408 dis

6   69K4238     DAIGLE
    45 29  9.961951   -72 31 38.850628       247.350000
   0.2953850819E-05   0.0000000000E+00   0.0000000000E+00
       417 dis         0             0
   0.2575428970E-05   0.0000000000E+00   0.0000000000E+00
       310 dis         0             0
   0.2161766696E-05
       310 dis

7   712051      ST MAJORIQUE
    45 54 58.861380   -72 38  1.304256       52.908000
   0.7749727553E-05   0.0000000000E+00   0.0000000000E+00
       417 dis         0             0
   0.9316988431E-05   0.0000000000E+00   0.0000000000E+00
       401 dis         0             0
   0.5348008518E-05
       401 dis

8   712056      WICKHAM
    45 46  7.554535   -72 36 21.129165       77.103000
   0.7476142592E-05   0.0000000000E+00   0.0000000000E+00
       417 dis         0             0
   0.2536399125E-05   0.0000000000E+00   0.0000000000E+00
         9 dir         0             0
   0.1995342595E-05
       310 dis

9   08207       OWLS HEAD
    45  3 45.158627   -72 17 52.885732       722.260000
   0.2594546255E-05   0.0000000000E+00   0.0000000000E+00
       216 dir         0             0
   0.1615778936E-05   0.0000000000E+00   0.0000000000E+00
       215 dir         0             0
   0.1226679828E-05
       408 dis

10  09202       DUSABLE
    46 12 37.072604   -73 11 59.806107       104.781000
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        0.3101339429E-05   0.0000000000E+00   0.0000000000E+00
             225 dir        0             0
        0.3994191380E-05   0.0000000000E+00   0.0000000000E+00
             224 dir        0             0
       -0.4188316514E-05
             232 dir

 11     09204        CARMEL
         46 29 58.619050   -72 37 39.231578       158.621000
        0.2303922089E-05   0.0000000000E+00   0.0000000000E+00
             225 dir        0             0
        0.4533089469E-05   0.0000000000E+00   0.0000000000E+00
             224 dir        0             0
        0.4979524888E-05
             224 dir

 12     09205        HAM
         45 47 28.152005   -71 38  0.762182       683.980000
        0.2419608259E-05   0.0000000000E+00   0.0000000000E+00
             216 dir        0             0
        0.2157032141E-05   0.0000000000E+00   0.0000000000E+00
             224 dir        0             0
        0.2422139615E-05
             224 dir

 13     09207        HEREFORD
         45  4 57.225000   -71 36  3.592868       845.530000
        0.2897477348E-05   0.0000000000E+00   0.0000000000E+00
             216 dir        0             0
        0.6010754979E-06   0.0000000000E+00   0.0000000000E+00
             109 dir        0             0
        0.7700208902E-06
             393 dis

 14     09208        MEGANTIC
         45 26 51.274493   -71  7 13.028206       1059.467500
        0.2854532175E-05   0.0000000000E+00   0.0000000000E+00
             216 dir        0             0
        0.3987041417E-05   0.0000000000E+00   0.0000000000E+00
             226 dir        0             0
        0.3929598540E-05
             375 dis

 15     09209        THETFORD
         46  8 48.515181   -71 20 11.437419       666.570000
       -0.8307484869E-05   0.0000000000E+00   0.0000000000E+00
             226 dir        0             0
        0.9124208779E-05   0.0000000000E+00   0.0000000000E+00
             226 dir        0             0
        0.1011247600E-04
             375 dis

 16     09210        LINIERE
         45 49 45.115005   -70 22 20.319318       750.290000
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     0.2999085842E-05   0.0000000000E+00   0.0000000000E+00
          216 dir          0              0
     0.5236910186E-05   0.0000000000E+00   0.0000000000E+00
          229 dir          0              0
     0.5514152198E-05
          229 dir

17   09216      ARTHABASKA
      46  3 14.104500   -71 53 16.799699       321.731000
     0.1191938293E-04   0.0000000000E+00   0.0000000000E+00
          412 dis          0              0
     0.1587670111E-04   0.0000000000E+00   0.0000000000E+00
          412 dis          0              0
     0.1798636910E-04
          412 dis

18   14200      STRATFORD
      45 47 39.761115   -71 15 20.039726       409.870000
    -0.1980789843E-04   0.0000000000E+00   0.0000000000E+00
          228 dir          0              0
     0.1777116246E-04   0.0000000000E+00   0.0000000000E+00
          228 dir          0              0
     0.1534542300E-04
          422 dis

19   65K0335     CROIX
      45 33 46.497791   -70 52 22.787639       464.720000
     0.0000000000E+00   0.0000000000E+00   0.0000000000E+00
          0               0              0
     0.0000000000E+00   0.0000000000E+00   0.0000000000E+00
          0               0              0
     0.0000000000E+00
          0

20   66KP115     CARIBOU E-15
      46  0 15.971155   -71 24 10.097848       529.160000
    -0.5562603072E-05   0.0000000000E+00   0.0000000000E+00
          136 dir          0              0
     0.7851738514E-05   0.0000000000E+00   0.0000000000E+00
          156 dir          0              0
     0.5604775540E-05
          391 dis

21   68K2071     SHERBROOKE
      45 20 46.266313   -71 55 33.826884       412.850000
     0.2689331661E-05   0.0000000000E+00   0.0000000000E+00
          216 dir          0              0
     0.8202859858E-06   0.0000000000E+00   0.0000000000E+00
           88 dir          0              0
     0.7052087482E-06
          338 dis

22   68K2073     BEAUVOIR
      45 27 17.108246   -71 53 53.751099       280.300000
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```
       0.3135858519E-05   0.0000000000E+00   0.0000000000E+00
            47 dir          0            0
       0.2486786173E-05   0.0000000000E+00   0.0000000000E+00
            83 dir          0            0
       0.2375535604E-05
           358 dis

23     692009     HONORE
        45 56 53.174517   -70 50 16.176131      447.770000
       0.0000000000E+00   0.0000000000E+00   0.0000000000E+00
            0          0            0
       0.0000000000E+00   0.0000000000E+00   0.0000000000E+00
            0          0            0
       0.0000000000E+00
            0

24     692010     GRELOTS
        45 59  2.203154   -71  1 21.673533      381.909000
      -0.7953237833E-05   0.0000000000E+00   0.0000000000E+00
           152 dir          0            0
       0.5792329569E-05   0.0000000000E+00   0.0000000000E+00
           158 dir          0            0
       0.3850007066E-05
           158 dir

25     692011     BROUGHTON
        46  8 17.589232   -71  5 49.617509      581.498000
       0.0000000000E+00   0.0000000000E+00   0.0000000000E+00
            0          0            0
       0.0000000000E+00   0.0000000000E+00   0.0000000000E+00
            0          0            0
       0.0000000000E+00
            0

26     692012     ADSTOCK
        46  1 46.979836   -71 12 18.397682      685.978000
      -0.1230437573E-04   0.0000000000E+00   0.0000000000E+00
           151 dir          0            0
       0.9751390846E-05   0.0000000000E+00   0.0000000000E+00
           151 dir          0            0
       0.1287876464E-04
           395 dis

27     69K4239    DUSSAULT
        45 28  4.318988   -72 13 51.763295      402.500000
       0.4064810816E-05   0.0000000000E+00   0.0000000000E+00
            47 dir          0            0
       0.1295113346E-05   0.0000000000E+00   0.0000000000E+00
            58 dir          0            0
       0.1122547988E-05
           308 dis

28     69K4240    GALLUP HILL
        45 38  6.697228   -72 11 57.261594      319.590000
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        0.4348054981E-05   0.0000000000E+00   0.0000000000E+00
             47 dir          0                0
        0.4027722157E-05   0.0000000000E+00   0.0000000000E+00
            417 dis          0                0
        0.4826614114E-05
            417 dis

 29     69K4241      SOUTH DURHAM
         45 38 48.365666   -72 21 26.900929       179.840000
        0.5526232020E-05   0.0000000000E+00   0.0000000000E+00
             47 dir          0                0
        0.4290238242E-05   0.0000000000E+00   0.0000000000E+00
            310 dis          0                0
        0.3002360240E-05
            310 dis

 30     69K4242      PINNACLE
         45 43 21.443910   -72  0 41.610936       388.010000
        0.3371893597E-05   0.0000000000E+00   0.0000000000E+00
             47 dir          0                0
        0.2236453538E-05   0.0000000000E+00   0.0000000000E+00
            416 dis          0                0
        0.1793648331E-05
            416 dis

 31     69K4243      LAROCHELLE
         45 31 43.227302   -72  4 23.562369       304.990000
        0.3266237003E-05   0.0000000000E+00   0.0000000000E+00
             47 dir          0                0
        0.1435730976E-05   0.0000000000E+00   0.0000000000E+00
             75 dir          0                0
        0.1486183689E-05
            350 dis

 32     69K4346      CHARLES
         45 52 34.266330   -72 27 39.849084        63.770000
        0.7867890981E-05   0.0000000000E+00   0.0000000000E+00
             11 dir          0                0
        0.2237129517E-05   0.0000000000E+00   0.0000000000E+00
            317 dis          0                0
        0.2443078905E-05
            317 dis

 33     69K4348      LEMAIRE
         45 51 23.401352   -72 34 52.406629        63.700000
        0.7749620783E-05   0.0000000000E+00   0.0000000000E+00
            417 dis          0                0
        0.1378370310E-05   0.0000000000E+00   0.0000000000E+00
            317 dis          0                0
        0.2543066967E-05
            317 dis

 34     69K4349      BREBOEUF
         45 50 20.776812   -72 29 59.688552        59.560000
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     0.7749624063E-05   0.0000000000E+00   0.0000000000E+00
         417 dis        0           0
     0.1378626132E-05   0.0000000000E+00   0.0000000000E+00
         317 dis        0           0
     0.2543066967E-05
         317 dis

35   69K4350     HEMMING
      45 51 46.540782   -72 27  0.785585      85.800000
     0.3299870489E-04   0.0000000000E+00   0.0000000000E+00
         323 dis        0           0
     0.3869778647E-04   0.0000000000E+00   0.0000000000E+00
         323 dis        0           0
     0.2082584995E-04
          34 dir

36   70K4244     MAGOG
      45 13 57.380010   -72  7  2.328885     317.400000
     0.3125550595E-05   0.0000000000E+00   0.0000000000E+00
          48 dir        0           0
     0.1775246505E-05   0.0000000000E+00   0.0000000000E+00
         366 dis        0           0
     0.1629930264E-05
         296 dir

37   70K4245     AUSTIN
      45 12  7.762168   -72 14 44.998966     290.520000
    -0.3892163923E-05   0.0000000000E+00   0.0000000000E+00
          53 dir        0           0
     0.3882404481E-05   0.0000000000E+00   0.0000000000E+00
          53 dir        0           0
     0.3175721289E-05
         365 dis

38   70K4631     HATLEY
      45  9  8.363523   -71 53 18.366476     395.940000
     0.2849218070E-05   0.0000000000E+00   0.0000000000E+00
         216 dir        0           0
     0.1392014735E-05   0.0000000000E+00   0.0000000000E+00
         109 dir        0           0
     0.1255207336E-05
         330 dis

39   70K4632     MARTIN
      45 18 23.810822   -71 38 31.356562     396.220000
     0.2907676552E-05   0.0000000000E+00   0.0000000000E+00
         216 dir        0           0
     0.1293804221E-05   0.0000000000E+00   0.0000000000E+00
         108 dir        0           0
     0.1238682002E-05
         393 dis

40   70K4633     CHAPMAN
      45 34 16.985548   -71 40 44.175426     630.990000
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0.2809255876E-05   0.0000000000E+00   0.0000000000E+00
     47 dir           0            0
0.1139954816E-05   0.0000000000E+00   0.0000000000E+00
     393 dis          0            0
0.1463766489E-05
     393 dis

41   70K4634    ASBESTOS
   45 45 16.869121   -71 54 42.797011      309.870000
  -0.6302590378E-05   0.0000000000E+00   0.0000000000E+00
     418 dis          0            0
   0.4380594903E-05   0.0000000000E+00   0.0000000000E+00
     418 dis          0            0
   0.2612630896E-05
     412 dis

42   70K4635    WEEDON
   45 38 32.848965   -71 26 42.228373      386.480000
   0.2944282103E-05   0.0000000000E+00   0.0000000000E+00
     216 dir          0            0
   0.1462416377E-05   0.0000000000E+00   0.0000000000E+00
     393 dis          0            0
   0.1634337190E-05
     393 dis

43   70K4637    GILBERT
   45 36 21.037028   -70 58 44.422322      544.460000
  -0.1065074819E-04   0.0000000000E+00   0.0000000000E+00
     277 dir          0            0
   0.1350012469E-04   0.0000000000E+00   0.0000000000E+00
     277 dir          0            0
   0.7872459230E-05
     426 dis

44   70K4638    COULOMBE
   45 51 12.417696   -71 29 19.031780      438.270000
   0.2935917950E-05   0.0000000000E+00   0.0000000000E+00
     216 dir          0            0
   0.2318051541E-05   0.0000000000E+00   0.0000000000E+00
     136 dir          0            0
   0.2066990702E-05
     384 dis

45   70K4639    MOISAN
   45 53 56.871982   -71 34 35.977949      569.740000
  -0.8540689454E-05   0.0000000000E+00   0.0000000000E+00
     141 dir          0            0
   0.8638607481E-05   0.0000000000E+00   0.0000000000E+00
     121 dir          0            0
   0.7271839907E-05
     378 dis

46   712050     BON CONSEIL
   45 58 40.678773   -72 22 58.101451       91.631000
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     0.7749729450E-05   0.0000000000E+00   0.0000000000E+00
         417 dis          0            0
     0.9320161087E-05   0.0000000000E+00   0.0000000000E+00
         401 dis          0            0
     0.5348008518E-05
         401 dis

47   712053     ST ZEPHIRIN
      46  4 47.569228   -72 39  2.842686        25.023000
     -0.1111213491E-04   0.0000000000E+00   0.0000000000E+00
         401 dis          0            0
      0.2135469992E-04   0.0000000000E+00   0.0000000000E+00
         401 dis          0            0
      0.1695259473E-04
         401 dis

48   712055     MALLARD
      45 46 28.667244   -72 24  5.686794       142.128000
      0.5503102695E-05   0.0000000000E+00   0.0000000000E+00
          47 dir          0            0
      0.4757724749E-05   0.0000000000E+00   0.0000000000E+00
         417 dis          0            0
      0.4607355053E-05
         417 dis

49   712057     DRUMMOND
      45 54 16.662029   -72 31 35.443646        63.880000
      0.7752047465E-05   0.0000000000E+00   0.0000000000E+00
         417 dis          0            0
      0.1308349826E-05   0.0000000000E+00   0.0000000000E+00
         400 dis          0            0
      0.1457931203E-05
         317 dis

50   71K6154    STORNOWAY
      45 42 45.387397   -71 12 13.529074       483.170000
     -0.4225622773E-05   0.0000000000E+00   0.0000000000E+00
         146 dir          0            0
      0.4797120232E-05   0.0000000000E+00   0.0000000000E+00
         146 dir          0            0
      0.2794709671E-05
         386 dis

51   71K6155    STE PRAXEDE
      45 53 51.218892   -71 13 23.166549       364.290000
     -0.5422173217E-05   0.0000000000E+00   0.0000000000E+00
         146 dir          0            0
      0.3967224861E-05   0.0000000000E+00   0.0000000000E+00
         151 dir          0            0
      0.3178432040E-05
         386 dis

52   71K6156    SEBASTIEN
      45 45 36.572370   -70 55 19.585027       799.230000
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      -0.9159936778E-05   0.0000000000E+00   0.0000000000E+00
            146 dir          0              0
       0.8023707250E-05   0.0000000000E+00   0.0000000000E+00
            146 dir          0              0
       0.6881894531E-05
            386 dis

 53   71K6159     LAPOINTE
       45 54  6.374812   -71 34 14.982987       600.040000
      -0.5360384814E-05   0.0000000000E+00   0.0000000000E+00
            383 dis          0              0
       0.9092228940E-05   0.0000000000E+00   0.0000000000E+00
            383 dis          0              0
       0.1107144575E-04
            383 dis

 54   71K6165     VIANNEY
       46  4 52.998888   -71 37 30.248075       564.280000
      -0.7362515097E-05   0.0000000000E+00   0.0000000000E+00
            245 dir          0              0
       0.1502158604E-04   0.0000000000E+00   0.0000000000E+00
            383 dis          0              0
       0.2253310123E-04
            383 dis

 55   72K7455     VICTORIAVILLE
       46  0 41.601241   -71 55 46.779541       246.133000
       0.0000000000E+00   0.0000000000E+00   0.0000000000E+00
             0            0             0
       0.0000000000E+00   0.0000000000E+00   0.0000000000E+00
             0            0             0
       0.0000000000E+00
             0

 56   72K7457     SEVIGNY
       45 56 13.678515   -71 43 17.746867       500.100000
       0.6385320893E-05   0.0000000000E+00   0.0000000000E+00
            383 dis          0              0
       0.9078455909E-05   0.0000000000E+00   0.0000000000E+00
            412 dis          0              0
       0.9200836578E-05
            412 dis

 57   72K7462     KINGSEY FALLS
       45 54  5.080173   -72  4 40.320796       125.670000
       0.1617695032E-04   0.0000000000E+00   0.0000000000E+00
            412 dis          0              0
       0.1396636558E-04   0.0000000000E+00   0.0000000000E+00
            412 dis          0              0
       0.1159093134E-04
            244 dir

 58   72K7463     ST FELIX
       45 47 58.863308   -72 11 28.930794       180.680000
```

```
   -0.6568929503E-05   0.0000000000E+00   0.0000000000E+00
        418 dis         0            0
    0.8513433793E-05   0.0000000000E+00   0.0000000000E+00
        418 dis         0            0
    0.7365212531E-05
        418 dis
99
```

8. PROPOSED SPECIFICATIONS FOR THE TOTAL ANALYSIS OF NETWORKS

8.1 Overall Scheme

Based upon the scientific analysis presented in the foregoing chapters, we are now in a position to propose a methodology to be used in the complete analysis of a 2D network (Table 8.1). The columns contain the various quantities to be assessed, while the rows contain the various measures and tests to be used. The proposal clearly integrates the standard assessment tools of random error analysis (covariance analysis — row 1), with that of the robustness (row 2), and external reliability (row 3) analyses. The quantities to be assessed consist of the estimated positions, model, observables, and functions of estimated positions. The observation and model measures are used in two modes: preanalysis and postanalysis.

8.2 Preanalysis

In standard statistical testing procedures, it is mandatory to predict beforehand the point and relative confidence regions of the coordinates. This yields a measure of how random errors will propagate from the observations into the estimated positions.

It is also mandatory to predict how the systematic blunders (if made) will propagate throughout the network and bias the estimated positions. These can be measured by internal reliability measures on the observables, i.e., the maximum undetectable errors, robustness, and external reliability measures on the estimated positions. Here, the robustness analysis gives us a measure of how strong the network is in resisting blunders or systematic errors in the observations. Recall, that the internal reliability measure is an estimate of how large a blunder can be before standard statistical testing can catch it, whereas robustness and the external reliability quantify the effect on the unknown parameters.

Table 8.1 Total analysis of a network.

Quantity Assessed / Type of Measure	Observations and model: Preanalysis	Observations and model: Postanalysis	Positions pre- and postanalysis	Network as a whole	Functions of positions	Remarks
Accuracy (Type I error — random)	Given C_ℓ and design matrices **A** and **B**, predict $C_{\hat{x}}$ $\Rightarrow$ (1-α) confidence ellipses	Tests listed in Tables 13.2 to 13.5 in Vanicek and Krakiwsky [1986]	Point and relative (1-α) confidence ellipses	a) Test on $\hat{\sigma}_0^2 / \sigma_0^2$ b) Test on distribution of residuals	a) Accuracy of position differences b) Accuracy of distances c) Accuracy of directions	These are all standard tests and measures
Internal Reliability, Robustness (Type II error — blunder)	a) Maximum undetectable errors $\nabla\ell_i$ b) Redundancy numbers r_i c) Predicted residuals $v_i = r_i \nabla\ell_i$	a) Max. undetectable errors $\hat{\nabla}\ell_i$ b) Redundancy numbers r_i c) Actual residuals $\hat{v}_i = r_i \hat{\nabla}\ell_i$	a) Robustness in scale σ b) Robustness in shear γ c) Robustness in twist ω	a) Average $\lvert\sigma\rvert$ b) Average γ c) Average $\lvert\omega\rvert$	a) Strain of a line b) Rotation of a line	Robustness of functions to be yet formulated
External Reliability (Type II error — blunder)			External reliability ∇x_i	a) Global external reliability $\nabla \mathbf{x}^T C_{\hat{x}}^{-1} \nabla \mathbf{x}$ b) Average ext. reliability ∇x_i		Only under exceptional circumstances

8.3 Postanalysis

Postanalysis, like preanalysis, is extended to take care of the Type II error, that is, the quantification of what happens when one considers the presence of blunders in the solution. The standard tests consisting of the null hypothesis H_0 (no blunders) must be amended to include the alternative hypothesis H_a (blunders). In this way, we are able to track down how our tests are affected by this new dimension.

The tests affected by the consideration of H_a are those listed in Vaníček and Krakiwsky [1986] as follows:

(a) Univariate testing of an observational series as a unit (Table 13.2).

(b) Univariate testing of individual observations (Table 13.3).

(c) Multivariate testing of observables as model as a unit (Table 13.4).

(d) Multivariate testing of individual observables (Table 13.5).

8.4 Other Considerations

The proposed total analysis scheme includes an extended preanalysis activity where both the Types I and II errors are modelled. We note that for reasons explained in Chapter 5, the external reliability measures should be used only when robustness cannot be computed because of some peculiar network configuration.

Robustness of functions of estimated positions, such as computed distances, angles, possibly coordinate differences, have not been formulated yet. It is clear from the theoretical viewpoint that such measures should exist, but mathematical expressions for these are yet to be derived.

We reiterate the point here that robustness of a network has to be measured by three independent primitives. It is not possible to combine these into a single measure. Tolerance limits and design criteria for robustness will have to be worked out on the basis of a 'reasonable' selection of β_0-value (probability of Type II error). This selection requires further investigation.

9. CONCLUSIONS, RECOMMENDATIONS, AND ACKNOWLEDGEMENTS

The collaboration of UNB and U of C researchers on the comparison of reliability analysis with geometric strength analysis resulted in the conception of a new technique, robustness analysis, which is a natural merger of the two existing techniques. First experiences with robustness analysis show that it is a very powerful technique capable of providing a picture of the analysed network, which is complementary to the one furnished by the standard covariance analysis. 'Network robustness' (strength, as an ability to resist deformations induced by undetectable blunders, might be a term more readily understood) is invariant with respect to coordinate shifts and almost invariant with respect to orientation and scale changes.

Robustness is expressed in terms of three independent deformation primitives; namely, robustness in scale (strain), local configuration (shear), and twist (differential rotation). It thus makes no sense to talk about robustness in general but only about "robustness in scale," "robustness in shear," and "robustness in twist." This will sound complicated to a surveyor uninitiated in the concepts of deformation analysis, where the three primitives are used routinely. Let us emphasize here that the full description of a deformation cannot be achieved with fewer than three primitives. If we wish to deal with network strength meaningfully, then we have to accept this fact and learn to live with it. It seems to us that the introduction of robustness analysis will require some educational effort aimed at the surveying community. Specifically, a guide/manual will have to be written with the aim to assist in the transfer of knowledge.

We recommend that robustness analysis be used side-by-side with the standard covariance analysis for a complete network analysis in the future. The Canadian federal specifications for horizontal control networks should be extended to include robustness analysis. It should be mentioned here that under special circumstances, the 'external reliability' measure discussed in

Chapter 3 would have to be used (in case of geometrical singularity encountered at a network point or set of points) and provisions should be made for this in the specifications.

As we have seen in Chapter 7, it is not always easy, or even possible, to guess at the reason behind a specific weakness in the network from the network configuration alone. More experiments should be conducted with robustness analysis, and more experience gained with practical application as well as the interpretation of robustness analysis results, particularly before specific values of robustness tolerance limits can be imposed through federal specifications. Some general criteria, however, can be formulated already, and these were spelled out in Chapter 8. A better graphical representation of robustness primitives is a must. Our investigations were definitely hindered by the unavailability of a decent graphics package on the UNB Vax computer system.

A strategy will have to be worked out on how to deal with the two kinds of singularities that may arise in robustness analysis. While the generic singularity associated with the extreme weakness of the network has so far been shown by 'large' values of the robustness primitives, geometrical singularities have been simply eliminated by leaving out the singular points. More worrisome is the case of geometrical near-singularities such as the one encountered at station HEMMING in the analysis of the real network in Chapter 7. A measure of ill-conditioning based either on confidence regions for strength primitives or the value of the determinant in the least-squares fitting of planes in the determination of strain matrices will have to be devised.

Some refinement of the reliability analysis as the first part of robustness analysis is called for in order to understand better the role of the probabilities (significance levels) used in the univariate and multivariate tests and their impact on the non-centrality parameter λ_0. The total picture of how those probabilities work together should be assembled and illustrated on numerical examples to be shown in the guide/manual as mentioned above. Even though the appropriate selection of β_0-probability was not necessary in our investigations — β_0 affects only the scale of the robustness primitive plots — it will become necessary for formulating the robustness tolerance limits. This point thus deserves further investigation.

We wish to express our gratitude to Mr. M. Pinch, the scientific authority for the contract, for his cooperation, and for assembling for us the observations of an actual two-dimensional network used in our investigations. Mr. D. Beattie kindly helped us with some problems we had with the GHOST program. Ms. M. Wilson's assistance with various pieces of software on the UNB Department of Surveying Engineering Vax computer system is gratefully acknowledged. Ms. W. Wells has done her usual flawless editing and word processing of several iterations of this report.

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